CONFORMAL TENSORS AND CONNECTIONS

By OSWALD VEBLEN

DEPARTMENT OF MATHEMATICS, PRINCETON UNIVERSITY

Communicated August 15, 1928

In this note I attempt to set forth the system of invariants which is appropriate to the conformal geometry of Riemannian spaces. It is a development in the direction of my recent paper on projective tensors and connections[1] of the ideas established by T. Y. Thomas[2,3] and J. M. Thomas[4,5] in four papers in these PROCEEDINGS. A class of invariants is found which I call *conformal tensors* which have $(n + 2)^k$ components in each coördinate system and a linear law of transformation with coefficients determined by a certain group which I call the *enlarged conformal group*. This group has to the conformal group much the same relation that the affine group has to the Euclidean group.

It does not seem to be possible to find an invariant with $(n + 2)^3$ components which will play the rôle of an affine connection in covariant differentiation. But by using one with $(n + 2)^2 (n + 1)$ components analogous to that already used by T. Y. Thomas[3] and to the set of $(n + 2)^2 n$ components used by J. A. Schouten[6] it is possible to find most of the components of a "conformal derivative" of any conformal tensor and then to determine the remaining components by imposing an invariant condition on the tensor sought. We thus have a recursion process which generates an infinite sequence of conformal tensors from any given conformal tensor. Thus to get a complete sequence of invariants for a conformal geometry it will be sufficient to start with the conformal curvature tensor (whose components include those of Weyl's[7] conformal curvature tensor) and apply this process of conformal differentiation. These invariants would seem to be quite suitable for the complete development of the theory of conformal geometry outlined from a somewhat different point of view by E. Cartan.[8]

1. *The Fundamental Relative Tensor.*—According to an observation by T. Y. Thomas a conformal geometry of n dimensions is really the theory of a relative tensor of weight $-2/n$. We start with a family of Riemannian metrics all of which determine the same angle at each point. The formula for the differential of arc of these metrics is

$$ds^2 = \sigma g_{ij}\, dx^i\, dx^j \tag{1.1}$$

in which σ is an arbitrary scalar and g_{ij} an absolute covariant tensor of the second order. In a given coördinate system, there is a unique one of these metrics, namely,

$$ds^2 = \frac{g_{ij}}{g^{\frac{1}{n}}} dx^i dx^j = G_{ij} dx^i dx^j \tag{1.2}$$

for which the determinant of the coefficients is unity. The invariant which has these coefficients as its components in each coördinate system is a relative tensor of weight $-2/n$, that is to say it has the law of transformation

$$\overline{G}_{ij} = u^{-\frac{2}{n}} G_{ab} u^a_i u^b_j \tag{1.3}$$

in which

$$u^i_j = \frac{\partial x^i}{\partial \bar{x}^j} \text{ and } u = \left|\frac{\partial x}{\partial \bar{x}}\right|. \tag{1.4}$$

With it there is associated the relative contravariant tensor of weight $2/n$,

$$G^{ij} = g^{\frac{1}{n}} g^{ij}$$

such that

$$G^{ij} G_{ik} = \delta^j_k.$$

The determinant $|G^{ij}|$ is, of course, equal to unity.

2. *The Conformal Connection* is an invariant introduced by J. M. Thomas[4] which has the law of transformation,

$$\overline{K}^i_{jk} = K^a_{bc} v^i_a u^b_j u^c_k + \frac{\partial u^a_j}{\partial \bar{x}^k} v^i_a - \frac{1}{n} (\delta^i_j u^o_k + \delta^i_k u^o_j - \overline{G}_{jk} \overline{G}^{ia} u^o_a) \tag{2.1}$$

in which

$$u^o_j = \frac{\partial \log u}{\partial \bar{x}^j} \text{ and } v^i_j = \frac{\partial \bar{x}^i}{\partial x^j}.$$

It is connected with the tensor G_{ij}, as was remarked by T. Y. Thomas, by the same formula as defines the Chrystoffel symbols of the second kind

$$K^i_{jk} = \frac{1}{2} G^{ia} \left(\frac{\partial G_{aj}}{\partial x^k} + \frac{\partial G_{ak}}{\partial x^j} - \frac{\partial G_{jk}}{\partial x^a}\right). \tag{2.2}$$

In the Euclidean case its components vanish in all coördinate systems for which the components of the tensor G_{ij} are constants. The group of all transformations between these coördinate systems is the *enlarged conformal group*. It contains the conformal groups, each of which leaves a set of constant G's unaltered, as a family of mutually conjugate subgroups. Its relations with the conformal group are thus analogous to those of the affine group with the Euclidean group.

The theory of a conformal connection, i.e., of an invariant with the law of transformation (2.2) is a distinct geometry which may be called the *enlarged conformal geometry*.

The equations of the enlarged conformal group are the solutions of the differential equations obtained by setting the quantities K and $\overline{K}$ in (2.1) equal to zero. This is analogous to what happens in the affine and projective geometries; if we apply the same process to the affine and projective connections, respectively, we find the affine and the projective groups, respectively. The only difference is that the equations of the enlarged conformal group are more complicated. Before deriving them, let us find the equations for the conformal groups.

3. *The Conformal Group.*—A Euclidean conformal group is the group of all transformations which do not alter the components G_{ij} of a relative tensor of weight $-2/n$, provided these components are constants.

Such transformations can be set up by introducing coördinates analogous to tetracyclic coördinates. Let

$$x^{n+1} = \frac{1}{2} G_{ij} x^i x^j \tag{3.1}$$

and let $z^0, z^1, \ldots, z^{n+1}$ be homogeneous coördinates such that

$$\frac{z^A}{z^o} = x^A \qquad (A = 1, 2, \ldots, n + 1).$$

The relation (3.1) may then be written in the form

$$G_{\alpha\beta} z^\alpha z^\beta = 0 \qquad (\alpha, \beta = 0, 1, \ldots, n + 1) \tag{3.2}$$

provided we agree that

$$G_{o(n+1)} = G_{(n+1)o} = -1,$$
$$G_{io} = G_{oi} = G_{(n+1)i} = G_{i(n+1)} = G_{oo} = G_{n+1, n+1} = 0 \tag{3.3}$$
$$(i = 1, 2, \ldots, n).$$

The matrix of the quantities $G_{\alpha\beta}$ is that given by T. Y. Thomas on page 358, Vol. **12** of these PROCEEDINGS.

Since we have to use indices running over three ranges, we adopt the conventions that: small Roman letters are used for indices on the range $1, 2, \ldots, n$; capital Roman letters on the range $1, 2, \ldots, n, n + 1$; and Greek letters on the range $0, 1, 2, \ldots, n + 1$.

The equation (3.2) is undisturbed by linear transformations

$$z^\alpha = f^\alpha_\beta \bar{z}^\beta \tag{3.4}$$

provided

$$hG_{\alpha\beta} = G_{\sigma\tau} f^\sigma_\alpha f^\tau_\beta = G_{ij} f^i_\alpha f^j_\beta - f^o_\alpha f^{n+1}_\beta - f^o_\beta f^{n+1}_\alpha \tag{3.5}$$

where h is any constant. But on forming the determinants of the quantities on the two sides of (3.5) we see that

$$h = |f^\alpha_\beta|^{\frac{2}{n+2}}$$

where $|f^\alpha_\beta|$ is the determinant of the $(n + 2)^2$ quantities f^α_β. The group defined by (3.4) and (3.5) is the Euclidean conformal group which has been studied by Darboux and others by the use of more special coördinates. It has a sub-group consisting of those transformations which leave the origin (i.e., the point $z^A = 0$) invariant. For these transformations,

$$f^A_o = 0,$$

and we may assume without loss of generality that

$$f^o_o = 1.$$

The conditions (3.5) then reduce to

$$f^{n+1}_j = 0 \qquad \text{and} \qquad h = f^{\frac{2}{n}}$$

where f is the n-rowed determinant

$$f = |f^i_j|, \tag{3.6}$$

and also to

$$G_{ij} = f^{-\frac{2}{n}} G_{pq} f^p_i f^q_j, \tag{3.7}$$

$$f^{n+1}_{n+1} = f^{\frac{2}{n}}, f^o = f^{-\frac{2}{n}} G_{pq} f^p_i f^q_{n+1}, 2f^o_{n+1} = f^{-\frac{2}{n}} G_{pq} f^p_{n+1} f^q_{n+1}. \tag{3.8}$$

4. *The Enlarged Conformal Group* is the group of all transformations between coördinate systems in which the components G_{ij} are constants. For our purposes we need consider only the sub-group G of transformations between coördinate systems of this type with the same origin. Given any such coördinate system, the components G_{ij} in this coördinate system are determined and thus the $n + 2$ homogeneous coördinates z are determined. If we also specify $n^2 + n$ constants f^i_A, such that $f \neq 0$ (cf. (3.6)), we obtain a matrix

$$\begin{matrix} 1 & f^0_1 & f^0_2 & \dots & f^0_n & f^0_{n+1} \\ 0 & f^1_1 & f^1_2 & \dots & f^1_n & f^1_{n+1} \\ 0 & f^2_1 & f^2_2 & \dots & f^2_n & f^2_{n+1} \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & f^n_1 & f^n_2 & \dots & f^n_n & f^n_{n+1} \\ 0 & 0 & 0 & \dots & 0 & f^{\frac{2}{n}} \end{matrix} \tag{4.1}$$

in which the elements of the first row are determined by the equations (3.8). The elements of this matrix are the coefficients of a transformation

$$z^\alpha = f^\alpha_\beta \bar{z}^\beta. \tag{4.2}$$

This transformation obviously carries the components $G_{\alpha\beta}$ into another set of constant components which satisfy the conditions (3.3) and is therefore a transformation of our group. The components of the relative tensor in the coördinate system $\bar{x}$ so defined are

$$\bar{G}_{ij} = f^{-\frac{2}{m}} G_{ab} f^a_i f^b_j. \tag{4.3}$$

It is important to observe that the components G_{ij} in the coördinate system x are implicit in (4.1). Thus there is a family of matrices (4.1) determined by each coördinate system in which the components of the relative tensor are constants.

The equations (3.8) can be converted by (4.3) into

$$f^j_{n+1} = \bar{G}^{pq} f^j_p f^o_q,\ 2f^o_{n+1} = \bar{G}^{pq} f^o_p f^o_q, \tag{4.4}$$

which shows that the elements of the second to the $(n + 1)$st columns of (3.1) can be chosen arbitrarily and the elements of the last column then determined.

If we combine (4.2) with a transformation

$$\bar{z}^\alpha = \bar{f}^\alpha_\beta \bar{\bar{z}}^\beta$$

determined by the numbers $\bar{G}_{ij}$ and an arbitrary set of $n^2 + n$ numbers $\bar{f}^i_A$ such that $\bar{f} \neq 0$, we find

$$z^\alpha = \bar{\bar{f}}^\alpha_\beta \bar{\bar{z}}^\beta$$

in which

$$\bar{\bar{f}}^\alpha_\beta = f^\alpha_\gamma \bar{f}^\gamma_\beta.$$

Moreover, $\bar{\bar{f}}^\alpha_\beta$ are the elements of a matrix analogous to (4.1) built out of the quantities G_{ij} and $\bar{\bar{f}}^i_A$ such that

$$\bar{\bar{f}}^i_A = f^i_k \bar{f}^k_A.$$

Thus any coördinate system z, obtained by first transforming z to $\bar{z}$ by a transformation with matrix of the type (4.1) and then transforming from $\bar{z}$ to $\bar{\bar{z}}$ with an analogous matrix, may be obtained directly by a transformation with matrix of type (4.1) from z to $\bar{\bar{z}}$. It is easily verified also that if $\bar{z}$ and $\bar{\bar{z}}$ are two coördinate systems obtained from z by transformations with matrices of type (4.1) the transformation from $\bar{z}$ to $\bar{\bar{z}}$ is of analogous type. Hence, all the coördinate systems obtainable from the z coördinate system by transformations (4.2) are related among themselves by transformations of analogous type. The group of all these transformations is obviously a sub-group of our group G. It is in fact identical with G, as can be verified by carrying out the solution of the differential equations obtained by setting K and $\bar{K}$ equal to zero in (2.1).

In terms of the variables $x^1, x^2, \ldots, x^n, x^{n+1}$, the transformations of our group take the form

$$x^A = \frac{f^A_B \bar{x}^B}{1 + f^o_A \bar{x}^A}. \tag{4.5}$$

In terms of the original variables $x^1, x^2, \ldots, x^n$, we have the transformations,

$$x^i = \frac{f^i_j \bar{x}^j + \frac{1}{2} f^i_{n+1} \bar{G}_{pq} \bar{x}^p \bar{x}^q}{1 + f^o_j \bar{x}^j + \frac{1}{2} f^o_{n+1} \bar{G}_{pq} \bar{x}^p \bar{x}^q}. \tag{4.6}$$

The one of the equations (4.5) for which $A = n + 1$ is

$$G_{ij} x^i x^j = \frac{\overset{n}{f^2} \bar{G}_{pq} \bar{x}^p \bar{x}^q}{1 + f^o_j \bar{x}^j + \frac{1}{2} f^o_{n+1} \bar{G}_{pq} \bar{x}^p \bar{x}^q} \tag{4.7}$$

which is a consequence of (4.6), (4.3) and (4.4).

Let us introduce the notation

$$x_i = G_{ij} x^j = \frac{\partial x^{n+1}}{\partial x^i}. \tag{4.8}$$

Then on differentiating (4.6) we find

$$\frac{\partial x^i}{\partial \bar{x}^j} = \frac{f^i_j + f^i_{n+1} \bar{x}_j - x^i (f^o_j + f^o_{n+1} \bar{x}_j)}{1 + f^o_A \bar{x}^A} \tag{4.9}$$

and see that the functional determinant of (4.6) is

$$\left|\frac{\partial x}{\partial \bar{x}}\right| = \frac{1}{(1 + f^o_A \bar{x}^A)^n} \begin{vmatrix} 1 & f^o_1 & f^o_2 \cdots f^o_n & f^o_{n+1} \\ x^1 & f^1_1 & f^1_2 \cdots f^1_n & f^1_{n+1} \\ x^2 & f^2_1 & f^2_2 \cdots f^2_n & f^2_{n+1} \\ \cdot & \cdot & \cdot \quad \cdot & \cdot \\ \cdot & \cdot & \cdot \quad \cdot & \cdot \\ \cdot & \cdot & \cdot \quad \cdot & \cdot \\ x^n & f^n_1 & f^n_2 \cdots n f^n_n & f^n_{n+1} \\ 0 & -\bar{x}_1 & -x_2 \cdots -\bar{x}_n & 1 \end{vmatrix} = \frac{f}{(1 + f^o_A \bar{x}^A)^n}. \tag{4.10}$$

It then follows by a rather direct calculation that for the transformations (4.6),

$$\left|\frac{\partial x}{\partial \bar{x}}\right|^{-\frac{2}{n}} G_{ij} \frac{\partial x^i}{\partial \bar{x}^p} \frac{\partial x^j}{\partial \bar{x}^q} = f^{-\frac{2}{n}} G_{ij} f^i_p f^j_q = \bar{G}_{pq}. \tag{4.11}$$

We also find

$$\frac{\partial \log \left|\frac{\partial x}{\partial \bar{x}}\right|}{\partial \bar{x}^j} = -n \frac{f^o_j + f^o_{n+1} \bar{x}_j}{1 + f^o_A \bar{x}^A}. \tag{4.12}$$

5. There is an obvious isomorphism between the sub-group of the enlarged conformal group which leaves one point invariant and the group of all analytic transformations. Let

$$x^i = u^i(\bar{x}) \tag{5.1}$$

be an arbitrary analytic transformation and let

$$f^i_j = \left(\frac{\partial x^i}{\partial \bar{x}^j}\right)_o, \qquad u = \left|\frac{\partial x}{\partial \bar{x}}\right|$$

$$f^\alpha_o = \delta^\alpha_o, \quad f^o_j = -\frac{1}{n}\left(\frac{\partial \log u}{\partial \bar{x}^j}\right)_o, \quad f^o_{n+1} = \frac{1}{2} \bar{G}^{ij} f^o_i f^o_j \tag{5.2}$$

$$f^{n+1}_j = 0, \quad f^{n+1}_{n+1} = \left(u^{\frac{2}{n}}\right)_o, \quad f^j_{n+1} \bar{G}^{pq} f^j_p f^o_q,$$

where a subscript zero after a parenthesis indicates that the quantity within the parentheses is evaluated for $x = \bar{x}_o$. Then if $\bar{x}_o$ is an arbitrary point, there is determined at $\bar{x}_o$ a transformation of the form (4.6), i.e., a transformation of the enlarged conformal group. Moreover, the formulas show that if the transformation (5.1) is of the type (4.6), the transformation determined by it according to our rule is (5.1) itself. Thus we have at each point a multiple isomorphism between the totality of analytic transformations of coördinates and the enlarged conformal group of a Euclidean space.

6. *Conformal Tensors.*—This isomorphism established, it is clear that conformal tensors can be defined with a law of transformation in which the quantities f^α_β at each point play the same rôle as the derivatives $\delta x^i/\delta x^j$ in the law of transformation of affine tensors. But instead of the quantities f^α_β, we shall use the following quantities which differ from them by constant factors:

$$u^i_j = \frac{\partial x^i}{\partial \bar{x}^j} \qquad u = \left|\frac{\partial x}{\partial \bar{x}}\right|$$

$$u^\alpha_o = \delta^\alpha_o, \quad u^o_j = \frac{\partial \log u}{\partial \bar{x}^j}, \quad u^o_{n+1} = \frac{1}{2} \bar{G}^{ij} u^o_i u^o_j \tag{6.1}$$

$$u^{n+1}_j = 0, \quad u^{n+1}_{n+1} = u^{\frac{2}{n}}, \quad u^j_{n+1} = \bar{G}^{pq} u^j_p u^o_q .$$

The set of quantities so defined are the elements of the matrix given by T. Y. Thomas on page 356, Vol. 12 of these PROCEEDINGS.

Let us denote the quantities analogous to u^α_β which are determined by the inverse of (5.1), by v^α_β. Then we have

$$u^\alpha_\beta \, v^\beta_\gamma = \delta^\alpha_\gamma$$

and

$$u^i_j \, v^j_k = \delta^i_k \, .$$

We now define *a relative conformal tensor of weight N as an invariant whose* components in any two coördinate systems x and $\bar{x}$ are related by the law of transformation

$$\bar{T}^{\alpha \dots \beta}_{\gamma \dots \delta} = u^N \, T^{\epsilon \dots \zeta}_{\sigma \dots \tau} \, u^\sigma_\gamma \, \dots u^\iota_\delta \, v^\alpha_\epsilon \dots \, v^\beta_\zeta .$$

So, as a special case, a contravariant conformal vector of weight zero has the law of transformation

$$\bar{V}^\alpha = V^\sigma \, v^\alpha_\sigma .$$

The fundamental relative tensor G_{ij} whose law of transformation is (1.3) determines by the equations (3.3) a conformal tensor $G_{\alpha\beta}$ of weight $-2/n$: its law of transformation is

$$\bar{G}_{\alpha\beta} = u^{-\frac{2}{n}} \, G_{\sigma\tau} \, u^\sigma_\alpha \, u^\tau_\beta .$$

The conformal tensors satisfy the same laws of combination as the ordinary (affine) tensors. In addition, there are special rules due to the special form of the matrix (4.1). For example, the component of a covariant tensor for which all the subscripts are zero is a scalar of the same weight as the given tensor. Also, the component of any contravariant tensor of weight N and degree p for which the superscripts are all $n + 1$ is a scalar of weight $N + 2p/n$.

In order to have the affine, projective and conformal tensors fit together into a harmonious theory, I think it advisable to change the definition of the projective tensors used in my previous paper[1] so that u^0_j shall be given the same meaning as in (6.1) above. This merely changes some of the constants in the law of transformation. But it makes it possible to state a number of relationships between the projective and conformal tensors in a simple manner. For example, the components of any conformal covariant vector with indices 0, 1, 2, . . ., n are the components of a projective covariant vector. It also brings our notation into agreement with that used by T. Y. Thomas and by writers on five-dimensional relativity.

7. *The Conformal Gradient.*—There is a process analogous to covariant differentiation by which any conformal tensor determines another con-

formal tensor with one subscript more. But this involves an additional device which did not appear in the affine or the projective cases. We can illustrate it by the case in which the given tensor is a relative scalar of weight N.

The law of transformation of this scalar is

$$\bar{T} = T\, u^N .$$

On differentiating this we find equations from which we can infer that the quantities

$$T_{\omega}, = \frac{\partial T}{\partial x^j}\, \delta^j_\omega + N\, T \delta^o_\omega$$

in which the subscript ω runs over the values 0, 1, 2, . . ., n are $n + 1$ of the components of a conformal tensor. On account of the condition

$$u^{n+1}_\omega = 0,$$

the law of transformation of these components does not involve the $(n + 1)$st component. But the law of transformation of the latter does involve the other components. The $(n + 1)$st component could therefore be assigned arbitrarily in one coördinate system. We shall determine it by requiring the conformal vector which we are seeking to satisfy the invariant condition,

$$G^{\alpha\beta}\, T_\alpha T_\beta = 0.$$

On expanding, this becomes

$$-2\, T_o T_{n+1} + G^{ij} T_i T_j = 0$$

which determines T_{n+1}. Hence, we can define a conformal vector by the formula,

$$T_{,\alpha} = \frac{\partial T}{\partial x^j}\, \delta^j_\alpha + NT\, \delta^o_\alpha + \frac{1}{2NT}\, G^{ij}\, \frac{\partial T}{\partial x^i}\frac{\partial T}{\partial x^j}\, \delta^{n+1}_\alpha. \qquad (7.1)$$

This invariant will be called the *conformal gradient* of the relative scalar T. The formula fails when $N = 0$.

8. *The Extended Conformal Connection.*—In order to generalize covariant differentiation to the other conformal tensors we need another invariant with which to eliminate the higher derivatives which appear in the law of transformation of the derivatives of a conformal tensor. This invariant can be obtained by direct formal generalization from the affine and projective cases. The law of transformation,

$$\bar{K}^\alpha_{\beta\omega} = K^\rho_{\sigma\psi}\, v^\alpha_\rho\, u^\sigma_\beta\, u^\psi_\omega + \frac{\partial u^\rho_\beta}{\partial \bar{x}^j}\, \delta^j_\omega\, v^\alpha_\rho \qquad (8.1)$$

is transitive, provided the superscript and the first subscript run over the values 0, 1, 2, . . ., $n + 1$ whereas the second subscript (ω, ψ) runs over the indices 0, 1, 2, . . ., n only. Hence, the $(n + 2)^2(n + 1)$ functions $K^{\alpha}_{\beta\omega}$ are the components of an invariant which we shall call an extended conformal connection. This name is justified by the fact that the n^3 components K^{i}_{jk} are the components of a conformal connection in the sense of §2, provided that the extended conformal connection satisfies the invariant conditions,

$$K^{\alpha}_{o\beta} = K^{\alpha}_{\beta o} = -\frac{1}{n}\delta^{\alpha}_{\beta} \quad \text{and} \quad K^{n+1}_{ij} = -\frac{1}{n}G_{ij}. \tag{8.2}$$

The law of transformation (8.1) is the same as that of the "associated connection" defined by T. Y. Thomas, Vol. **12** of these PROCEEDINGS, p. 357. It is also related to the conformal connection considered by Schouten.[6]

Given any conformal tensor of weight N let us differentiate its law of transformation and eliminate the second derivative between the resulting formula and the law of transformation (8.1) of the extended conformal connection. If the given tensor is a conformal covariant vector T_{α}, for example, the result of this elimination is the set of equations which state that

$$T_{\alpha,\omega} = \frac{\partial T_{\alpha}}{\partial x^j}\delta^j_{\omega} + NT_{\alpha}\delta^o_{\omega} - K^{\beta}_{\alpha\omega}T_{\beta}$$

are components of a covariant conformal tensor.

Here again the subscript ω runs only over the indices 0, 1, 2, . . ., n. In order to determine the remaining components of the tensor which we are seeking we employ the invariant condition

$$G^{\alpha\beta}T_{o,\alpha}T_{o,\beta} = 0$$

which gives the formula

$$2T_{o,o}T_{o,n+1} = G_{ij}T_{o,i}T_{o,j}$$

and determines the component $T_{o,n+1}$. We then employ the condition

$$G^{\alpha\beta}T_{o,\alpha}T_{\gamma,\beta} = 0$$

which determines the rest of the components, for it expands into

$$T_{o,o}T_{\gamma,n+1} = -T_{o,n+1}T_{\gamma,o} + G^{ij}T_{o,i}T_{\gamma,j}\,.$$

The conformal tensor $T_{\alpha,\gamma}$ which is thus defined we call the *conformal derivative* of T_{α}.

If the given tensor is a contravariant vector of weight N, we differentiate