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Kurt Marti
Yuri Ermoliev
Marek Makowski
(Editors)

Coping with Uncertainty

Robust Solutions

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Robust Solutions

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Preface

The aim of the series of workshops on “Coping with Uncertainty” (*CwU*) organized at IIASA, Laxenburg, Austria, has been to provide researchers and practitioners from different areas with an interdisciplinary forum for discussing various ways of dealing with uncertainties in diverse areas, including environmental and social sciences, economics, policy-making, management, and engineering. The workshops proved to be successful, especially in cross-disciplinary sharing methods, ideas, and open problems.

Science-based support for addressing the on-going global changes needs solutions for fundamentally new scientific problems, which in turn require new concepts and tools. A key issue concerns a vast variety of practically irreducible uncertainties, including potential extreme events of high multidimensional consequences, which challenge traditional models, and thus require new concepts and analytical tools. This type of uncertainty critically dominates, e.g., the climate change debates. In short, the dilemma is concerned with enormous costs versus massive uncertainties of potentially extreme impacts. Traditional scientific approaches usually rely on real observations and experiments. Yet no sufficient observations exist for new problems, and “pure” experiments, and learning by doing may be very expensive, dangerous, or simply impossible. In addition, the available historical observations are often contaminated by “experimentator”, i.e., past actions, and policies. The complexity of new problems does not allow us to achieve enough certainty just by increasing the resolution of models or by bringing in more links. They require explicit treatment of uncertainties using “synthetic” information composed of available “hard” data from historical observations, the results of possible experiments, and scientific facts as well as “soft” data from experts’ opinions, scenarios, stakeholders, and public opinion. As a result of all these factors, our assessment will always have poor estimates. Therefore, the role of science-based support for addressing the new problems increasingly changes from the traditional “deterministic predictions” analysis to the design of strategies that are robust against the involved uncertainties and risks.

This volume contains contributions based on selected presentation at the *CwU*2007 workshop. The workshop aimed at contributing to a better understanding between practitioners dealing with the safety of complex systems under uncertainty, and scientists working on either corresponding modeling approaches, or on methods that can be adapted for improving the understanding and management of

uncertainty. The focus of the *CwU* 2007 was on novel approaches to supporting robust decision-making and design, especially when uncertainty is irreducible, consequences might be enormous, and the decision process involves stakeholders with diverse interests. Presentations dealt with open problems in this field, limitations of known approaches, novel methods and techniques, or lessons from applications of various approaches. In particular, contributions on the following issues were presented:

- Modeling different types of uncertainty (probabilistic and non-probabilistic)
- The formulation of appropriate deterministic substitute problems for different types of uncertainty
- Robustness of efficient solutions with respect to inherent uncertainties
- Simulation tools (for optimal decision/design under uncertainty)
- Safety and security of humans, environment, and vital infrastructure facing catastrophe risks
- Lessons that can be learned from designing and operating highly reliable systems
- Downscaling and discounting methods for handling spatial and temporal scales
- Benefits and costs of (partial) postponing decisions (aimed at reducing uncertainties)
- Open problems in the adequate treatment of uncertainties
- Concrete applications in economics, finance, engineering, energy, population, air quality, climate change, ecology, forestry, and other environmental problems

The workshop was organized at IIASA in December 2007, jointly by:

- * IIASA – International Institute for Applied Systems Analysis, Laxenburg, Austria
- * Federal Armed Forces University Munich, Germany

The scientific Program Committee included: Yuri Ermoliev, IIASA, Laxenburg (A); Leen Hordijk, IIASA, Laxenburg (A); Marek Makowski, IIASA, Laxenburg (A); Kurt Marti, Federal Armed Forces University Munich (D); Gerhard I. Schuëller, University of Innsbruck (A).

The organizers gratefully acknowledge the support of:

- GAMM – International Association of Applied Mathematics and Mechanics, and
- IIASA – International Institute for Applied Systems Analysis

Their generous support enabled the participation of many researchers who otherwise could not have attended the Workshop.

This volume contains chapters based on selected presentations at the *CwU* 2009 and an introductory short summary of the key issues related to the robust solutions. The chapters are organized into the following four parts:

1. *Modeling of uncertainty* discusses descriptions of uncertainties of different types (probabilities, theory of evidence and possibility, imprecise probabilities, fuzzy sets and variables).

2. *Robust solutions under uncertainty* presents new approaches to discounting applied to evaluation of investments for catastrophic risk management, and to cost-effective and environmentally-safe emission trading under uncertainties, as well as modern quantitative modeling methodologies for analysis of network risks and design of robust networks under uncertainty.
3. *Analysis and optimization of technical systems and structures under uncertainty* deals with state estimation of dynamical systems in case of uncertainties of initial conditions and dynamic parameters described by means of certain ellipsoids, and with the derivation of stochastic linear programs for the reliability-based optimization of plane frames under stochastic uncertainty with respect to external loadings and material parameters.
4. *Analysis and optimization of economic and engineering systems under uncertainty* discusses the variability of the atmospheric deposition of nitrogen in the sea, the treatment of risks and uncertainties in planning agricultural production allocation and expansion, the uncertainty in greenhouse gas emission estimates, consequences of the weather forecasts for the optimal control of agricultural production, and the estimation error in retrieving carbon dioxide column abundances obtained from the GOSAT satellite.

We express our gratitude to all referees, and we thank all authors for the timely delivery of the final version of their contributions to this volume. Furthermore, we thank Ms Elisabeth Löbl of the Federal Armed Forces University Munich for her support in the preparation of this volume. Finally, we thank Springer-Verlag for including the Proceedings in the Springer Lecture Notes Series “LNEMS”.

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Kurt Marti
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Chapter 1

General Remarks on Robust Solutions

Y. Ermoliev, M. Makowski, and K. Marti

We summarize here the background and key concepts related to robust solutions in the context of supporting decision-making for problems characterized by deep uncertainties, which also were in the focus of the previous workshops on *Coping with uncertainty*, see, e.g., [3]. Although such problems are fundamentally different from statistical decision models, yet basic ideas of robust statistics are applicable to methods supporting robust decision-making under uncertainty. The main new issues are concerned with a proper representation of uncertainty, and its interactions with decisions. In particular, a key issue is the sensitivity of robust decisions with respect to low probability catastrophic events, that are of critical importance for analyzing global change problems. Robust decisions for problems exposed to extreme catastrophic events are essentially different from over-simplified decisions that ignore such events. Specifically, a proper treatment of extreme/rare events requires new paradigms of rational decisions, new performance indicators, and new spatio-temporal dimensions of heterogeneous interdependencies including network externalities and risks. This, in particular, needs new approaches to downscaling, upscaling and discounting.

Global change processes, in particular climate change, involve inherently unpredictable complex interactions between natural and human-created systems therefore proper modeling of these processes must rely on adequate treatment of uncertainties, and their effects on human's decisions. Traditional natural science models are based on relations whose validity is estimated from repetitive experiments and observations. If experiments do not affect the underlying relations, then repetitive observations allow to derive them by using the statistical decision theory. Unfortunately, human-created processes do not follow fixed relations. Elements of these processes change their dimensions and structure. For example, introduction of new

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technologies may increase or reduce uncertainties, risks, critical thresholds and discontinuities. Exact identification of global climate change processes is impossible because such processes are non-stationary, have delayed responses, and human or natural actions may have catastrophic irreversible consequences.

Under inherent uncertainty of heterogeneous processes the role of integrated models rests on the ability to guide comparative analysis of rational decisions. Although exact evaluations are impossible, the preference structure among decisions can be a stable basis for a relative ranking of alternatives in order to design robust policies, which must be in a sense optimal against all relevant uncertainties. It is commonly known that finding out (without exact measurement) which of two parcels is the heavier is much easier than evaluating weight of each parcel.

The term “robust” was introduced into statistics in 1953 by Box [2] and acquired recognition after the publication of a path-breaking paper by Huber [5]. As Huber admits, researchers had long been concerned with the sensitivity of standard estimation procedures to “bad” observations (outliers), and the word “robust” was loaded with many, sometimes inconsistent connotations, frequently for the simple reason of conferring respectability on it. Appeal for robustness [4] probably dates back to pre-history of statistics. A distant outlier in observations ruins the least square analysis, therefore rejection of outliers is a sort of robust statistical procedure. The discussion about the rejection of outliers is at least as old as the 1777 publications of Daniel Bernoulli [1]. The mean is not robust to outliers, whereas the median is robust. Therefore, switching from the mean to the median for long-tailed data increases robustness. This is also equivalent to switching from quadratic (least square) smooth optimization to non-smooth optimization principles.

According to Huber [5], “...any statistical procedure...should be robust in the sense that small deviations from the model assumptions should impair the performance only slightly”. This concept of robustness corresponds to standard mathematical ideas of continuity and stability: when disturbances become small, the performance of the perturbed model also deviates slightly. In other words, a robust procedure is in a certain sense optimal with respect to all uncertainties from a neighborhood of the model. Huber introduced rigorous notions of robustness based on probabilistic minimax approach and Choquet capacity (imprecise probabilities), which lead to specific non-smooth stochastic optimization models. By using appropriate neighborhoods of probability measures (e.g., with respect to ϵ -contaminated probabilities, Levy distance, or Kolmogorov distance), he derived robust estimators optimizing the worst that can happen over the neighborhood of the model with respect to a certain performance indicator. Neighborhoods of probability measures can also be characterized by Choquet capacities, i.e., functions which define sets of probability measures by taking all probabilities which lie below (or above) a capacity (point-wise).

These basic ideas of robust statistics as well as the infinitesimal robustness introduced by Hampel [4] are also used for more general decision problems under uncertainty. In particular, the infinitesimal approach is based on the fact that many statistics and solutions of general decision models can be considered as functionals in the space of probability measures. The robustness information is then provided

by the inference functions, roughly speaking, a derivative (in the space of probability measures) of a statistic or a performance indicator at an underlying distribution. There are also important concepts of Bayesian and non-Bayesian robustness, where we need not only robustness against deviations from the given parametric model, but also against uncertainties of the prior distributions.

The word “robust” has become fashionable in statistical decision theory and other disciplines dealing with data analysis and decisions [4], in particular, for dealing with questions such as: Which data are of critical importance and should be examined with a special care? What methods provide the greatest safety? How safe are results of a model that is known only approximately?

As the concept of statistical robustness is in a sense similar to the problem of local stability of dynamic systems, the robustness in deterministic control theory was introduced as an additional requirement on the stability of optimal trajectories. In other words, additional constraints were introduced in the form of a stability criterion. Optimization theory provides tools for analyzing and solving various decision making problems. For deterministic models robustness is defined similar to probabilistic minimax robustness in statistics: to optimize the worst that can happen to performance indicators over solutions x that satisfy feasibility constraints for all admissible values of uncertainty $\omega \in \Omega$. The set Ω is often characterized by a finite number of scenarios or simple sets such as intervals or ellipsoidal uncertainty which, in a sense, attempt to substitute for normal probability distributions in a simple but inconsistent with statistical analysis manner. It is clear that this type of deterministic worst-case robustness leads to extremely conservative decisions.

After the word robust become fashionable in statistics, it is being used in many senses, e.g., “quantitative robustness”, “ Π -robustness”, “B-robustness”, see, e.g., [4]. The situation becomes more complex for general decision problems under uncertainty dealing with quite different decision situations which may have a vast variety of different facets of robustness. Therefore, in order to avoid dangerous confusions, the term “robust” must be precisely defined in every specific context. This is similar with other notions whose meaning depends on the context, e.g., fairness, efficiency, optimality. For example, a decision is optimal only with respect to precisely specified conditions.

Statistical decision theory deals with situations in which a model of uncertainty and corresponding optimal solution are defined by a sampling model characterized by a probability measure P with an unknown vector of “true parameters” x^* . Vector x^* defines a desirable optimal solution, its performance can be observed from the sampling model, and the problem is to recover x^* from these data. Potential estimates of x^* define feasible solutions x of a statistical decision problem. It is essential that x does not affect the sampling model P so that the optimality and robustness of solutions can be evaluated by a distance from x^* by using its observable performance.

The general problems of decision making under uncertainty deal with fundamentally different situations. The model of uncertainty, feasible solutions, and performance of the optimal solution are not given and all of these have to be characterized from the context of the decision making situation, e.g., socio-economic,

technological, environmental, and risk considerations. As there is no information on true optimal performance, robustness cannot be also characterized by a distance from an observable true optimal performance. Therefore, the general decision problems may have rather different facets of robustness. In particular, probabilistic minimax solutions may seem too pessimistic or too optimistic for coping with potentially catastrophic events. Therefore, other concepts of robustness are required. Such concepts may be based on, e.g., expected utility maximization, stochastic optimization, stochastic minimax models.

In the presence of uncertainty, any related decision x results in multiple outcomes such as costs, benefits, damages, and risks, as well as indicators of fairness, equity, and environmental impacts. The outcomes depend not only on decisions x but also on uncertainty characterized by $\omega \in \Omega$, where Ω , denotes a set of admissible scenarios.

Scenario analysis is often used as a straightforward approach to find a decision that is “optimal” with respect to all scenarios by attempting to solve the decision problem for all possible scenarios. Unfortunately, a given decision x for different scenarios ω may have rather contradictory outcomes which do not really tell us which decision is reasonably good for all scenarios.

In 1738 mathematician Daniel Bernoulli (see discussion in [7]) introduced the concept of expected utility maximization as a rule for choosing decisions under multiple outcomes. It is assumed that all outcomes $g_i(x, \omega)$, $i = 1, \dots, K$ can be summarized in a single measure of preferability, e.g., a monetary payoff denoted by: $q(x, \omega) = Q(g_1, \dots, g_K)$. The standard expected utility model suggests to choose a decision x that maximizes an expected utility function

$$U(x) = Eu(q(x, \omega)) = \int u(q(x, \omega))P(d\omega),$$

i.e., in a sense, for all $\omega \in \Omega$, where $u(\cdot)$ is a utility associated with an aggregate outcome $q(x, \omega)$. The shape of u defines attitudes to risks. This model presupposes that one can rank the alternative scenarios ω according to weights – objective or subjective probability measure P .

The use of a probability measure as a degree of belief was formalized by Ramsey [6]. Savage published [7] a thorough treatment of expected utility maximization based on subjective probability as a degree of belief. As a result of this work the use of probability measure became a standard approach for modeling uncertainty by using “hard” observations and “soft” public and expert opinions in a consistent way within a single model. Although a decision maximizes the expected utility, in a sense, for all scenarios, still it cannot be considered to be a robust solution. The shortcomings of the expected utility model are well known. Generally, it is practically impossible to find a utility function that enables satisfactory aggregation of various attributes in one preferability measure, including attitudes to different risks, the distributional aspects of gains and losses, the rights of future generations, and responsibilities for environmental protection.

For complex problems it is natural that different performance indicators should be used to evaluate robustness of the integrated system in the same way as we use indicators of health (e.g., temperature and blood pressure for humans). An expected utility model is a specific case of stochastic optimization (STO) models that use various performance indicators $f_i(x, \omega)$, $i = 1, \dots, m$, one of which can be the expected utility (disutility). These indicators depend on outcomes $g_k(x, \omega)$, $k = 1, \dots, K$, on x and ω , i.e.,

$$f_i(x, \omega) := q_i(g_1, \dots, g_K, x, \omega).$$

A rather general STO problem is formulated as optimization (maximization or minimization) of the expectation function

$$F_0(x) = Ef_0(x, \omega) = \int f_0(x, \omega)P(d\theta)$$

subject to constraints

$$F_i(x) = Ef_i(x, \omega) = \int f_i(x, \omega)P(d\theta) \geq 0, \quad i = 1, \dots, m.$$

The choice of proper indicators $f_i(x, \omega)$ and outcomes $g_k(x, \omega)$, $k = 1, \dots, K$, is essential for the robustness of x . Globally or regionally aggregated outcomes are less uncertain but they may not reveal potentially dramatic heterogeneities induced by global changes on individuals, governments, and the environment. For instance, an aggregate income or growth indicators may not reveal an alarming gap between poor and rich regions, which may cause future instabilities.

By choosing appropriate outcomes $g_k(x, \omega)$ and functions $f_i(x, \omega)$, STO models allowing a natural and flexible way to represent various risks, abrupt changes, spatio-temporal heterogeneities, equity constraints and the sequential resolution of uncertainty in time. Often, under proper robustness requirements, $f_i(x, \omega)$ are analytically intractable, non-smooth, and even discontinuous functions, and probability measure P is chosen from a feasible set, thus is imprecise; moreover, P often depends on x , which is essential for modeling endogenous extreme events and catastrophic risks, and non-Bayesian robustness. It is often practically impossible to identify uniquely subjective (and objective) probability as a degree of beliefs. Most people cannot clearly distinguish between probability ranging roughly from 0.3 to 0.7. Decision analysis often has to rely on imprecise statements, for example, that event e_1 is more probable than event e_2 or that the probability p_1, p_2 of event e_1 or of event e_2 is greater than 50% and less than 90%. Therefore, feasible sets of probabilities may be represented by inequalities such as $p_1 \geq p_2, 0.5 \leq p_1 + p_2 \leq 0.9$. As in robust statistics, the robust solutions of general decision models can be derived by using worst-case (for a given decision) probability distribution from the feasible sets of distributions satisfying constraints of STO model. The applicability of the