A FIRST COURSE ON
ELECTRICAL DRIVES
(Second Edition)

S.K. Pillai
Professor
Department of Electrical Engineering
Indian Institute of Technology
Bombay

JOHN WILEY & SONS
New York Chichester Brisbane Toronto Singapore

First Published in 1989 by
WILEY EASTERN LIMITED
4835/24 Ansari Road, Daryaganj
New Delhi 110 002, India

Distributors:

Australia and New Zealand:
JACARANDA-WILEY LTD., JACARANDA PRESS,
JOHN WILEY & SONS, INC.
GPO Box 859, Brisbane, Queensland 4001, Australia

Canada:
JOHN WILEY & SONS CANADA LIMITED
22 Worcester Road, Rexdale, Ontario, Canada

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05–05 Block B, Union Industrial Building
37 Jalan Pemimpin, Singapore 2057

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WILEY EASTERN LIMITED
4835/24 Ansari Road, Daryaganj
New Delhi 110 002, India

North and South America and rest of the world:
JOHN WILEY & SONS, INC.
605 Third Avenue, New York, N.Y. 10158, USA

Library of Congress Cataloging in Publication Data

ISBN 0–470–21399–X John Wiley & Sons, Inc.
ISBN 81–224–0166–X Wiley Eastern Limited

Printed in India at Prabhat Press, Meerut. U.P.

Foreword

Electrical Drives play a vital role in engineering and industry both in this country and abroad. It is therefore essential that students of electrical engineering have a proper grounding in this subject. Conventional courses in Electrical Machines, however, are not adequate for the purpose as electric motors do not by themselves constitute an electrical drive and their characteristics have to be studied keeping in mind the types of control schemes (such as those using thyristor circuits) and the dynamics of the load. On the other hand, courses on 'Control Systems', 'Industrial Electronics' and 'Power Electronics' do not devote sufficient attention to electrical motor characteristics and mechanical load demands. It is thus necessary to have a course on the fundamentals of electrical drives, suitable for study by undergraduate students of electrical engineering. This book *A First Course on Electrical Drives*—is designed to meet the need for a textbook in English for such a course. The writing of the book has been supported by the Curriculum Development Cell of the Indian Institute of Technology, Bombay.

The book gives a comprehensive introduction to the dynamics of drives, the characteristics, starting and braking of dc and ac motors, as also their loading conditions, ratings and heating. There are separate chapters devoted to solid state controlled drives and industrial applications. The MKS system of units has been used throughout and Indian Standards Specifications have been adhered to. In addition to worked examples, most chapters include a number of problems designed to test the student's grasp of the subject.

The author, Dr. S.K. Pillai, has over twenty years experience of teaching and research in electrical engineering, and he has developed the material of this book over the past ten years while conducting lecture, tutorial and laboratory classes for final year undergraduate students of Electrical Engineering at the Indian Institute of Technology, Bombay. The style and organization of the work reflects the discerning insight of a teacher into the requirements of a student and each topic is developed step by step in a clear and cogent manner. I am confident, therefore, that this book will be welcomed by students and teachers alike.

April 16, 1982

R.E. BEDFORD
Dy. Director
Indian Institute of Technology
Bombay

Preface to the First Edition

Electrical drives offer a convenient means for controlling the operation of different equipment used in industry. The high reliability and great versatility of electrical drives, especially of those controlled by solid state devices, have resulted in their wide application. In fact, the growth and developments of electrical drives have been closely in parallel with those of automation in industry. In this context, it is indeed surprising that the theory and practice of electrical drives is even, at present, taught for Bachelor's degree students as one amongst various topics under the head "Utilization of Electrical Energy" and not as a separate subject, in many technical institutions in our country. One of the reasons may be the non-availability of suitable text books on the subject in English language. This volume is primarily written with the aim of providing a text book for the undergraduate students in Electrical Engineering on the fundamentals of electrical drives.

The contents of this book have gradually evolved over the last ten years from the notes used by the author in teaching final year undergraduate students of I.I.T. Bombay. A number of books and papers published, listed in the 'Bibliography' have been made use of in the preparation of this book. They not only indicate the extent of indebtedness of the author to those authors and publishers, but also provide the student with material for further reading. Most chapters include, in addition to a number of solved examples, problems to test how well the student has grasped the subject matter presented.

I gratefully acknowledge the assistance and encouragement given to me by several of my past students and colleagues. A special word of thanks is due to Prof. R.E. Bedford, who kindly agreed to write a Foreword for the book. I am very much indebted to the Curriculum Development Cell, I.I.T. Bombay for giving me the necessary financial support to prepare the first version of this book. Grateful acknowledgement is due to the Indian Standards Institution for granting permission to use the definition and the associated figures relevant to the different classes of duty indicated in I.S.S. 4722. I also wish to thank the production department of Wiley Eastern Limited for the great care with which they have worked in bringing the book to its present form. Last but not least, I wish to record my gratitude to my wife Vijaya and sons Sivan and Kumar for the sacrifices they have made that this work could be completed.

I.I.T. Bombay
April 1982

S.K. Pillai

Preface to the Second Edition

The present edition, essentially, introduces the students of Electrical Engineering to the recent additions in solid state controlled drives viz., brushless d.c. motors and switched reluctance motors.

In response to the comments made on the first edition by many teachers, a separate chapter on 'Stepper Motors' has been included. The drives used in the fast growing Indian Petroleum industry have also been discussed, in the chapter on 'Industrial Applications'.

The author would like to acknowledge the valuable advice and suggestions of many instructors and students who used the earlier edition.

I.I.T. Bombay S.K. PILLAI
August 1988.

Contents

Chapter 1

Introduction

1.1 Concept of an Electrical Drive

Most of the production equipment used in modern industrial undertakings consist of three important components, namely, the prime mover, the energy transmitting device and the actual apparatus or equipment that performs the desired job. The function of the first two components is to impart motion and operate the third one. The most commonly used prime mover is, of course, an electric motor, since it is far superior in performance to steam, hydraulic, diesel and other types of engines. Electric motors are, often, operated directly from a supply line, under their own inherent speed-torque characteristics and their operating conditions are dictated by the mechanical loads, connected to them. However, in many applications, the motors are provided with a control equipment by which their characteristics can be adjusted and their operating conditions with respect to the mechanical load varied to suit specific requirements. The most common control adjustment is of motor speed, but torque and acceleration or deceleration can also be adjusted. The control equipment usually consists of relays, contactors, master switches and solid state devices such as diodes, transistors and thyristors.

The aggregate of electric motor, the energy transmitting shaft and the control equipment by which the motor characteristics are adjusted and their operating conditions with respect to mechanical load varied to suit particular requirements, is called an *electrical drive*. The drive together with the load constitutes the drive system.

1.2 Classification of Electrical Drives

In general, electrical drives may be classified into three categories: Group drive, Individual drive and Multimotor drive.

Group drive consists of a single motor which actuates several mechanisms or machines by means of one or more line shafts supported on bearings. It is also called a line shaft drive. The line shaft fitted with multistepped pulleys and belts that connect these pulleys and the shafts of the driven machines serve to vary their speed.

Even after taking into account the cost of line shafts, pulleys, belts and other installations, the group drive is the most economic one, since the rating of the motor used may be comparatively less than the aggregate of ratings of the individual motors required to drive each equipment, because all of them may not be working simultaneously.

But, seldom is the group drive used, nowadays, due to the following disadvantages:

(a) Any fault that occurs in the driving motor renders all the driven equipment idle.
(b) Considerable power loss takes place in the energy transmitting mechanisms.
(c) Flexibility of layout of the different machines is lost, since they have to be located as to suit the layout of the line shaft.
(d) The use of line shaft, pulleys and belts make the drive untidy in appearance and less safe to operate.
(e) The level of noise produced at the worksite is quite high.

In the individual drive, an electric motor is used for transmitting motion to various parts or mechanisms belonging to a single equipment. For example, such a drive in a lathe rotates the spindle, moves the feed and also with the help of gears, imparts motion to the lubricating and cooling pumps of the lathe. In many applications, the individual drive consists of a motor, which is specially designed to form an integral part of the equipment.

In the case of individual drive too, the energy is transmitted to the different parts of the same mechanism by means of mechanical parts like gears, pulleys etc. Hence, there occurs some power loss. This disadvantage is removed in the case of multimotor drives.

In multimotor drives, separate motors are provided for actuating different parts of the driven mechanism. For example, in travelling cranes, there are three motors: one for hoisting, another for long travel motion and the third for cross travel motion. Paper mills, rolling mills, rotary printing machines, metal working machines etc. employ a large number of multimotor drives.

The use of individual drives and multimotor drives has enabled introduction of automation in production processes, which in turn has considerably increased the productivity of different industrial undertakings. Complete or partial automation helps to operate various mechanisms at optimum conditions and to increase reliability and safety of operations.

Chapter 2

Dynamics of Electrical Drives

The electromagnetic forces or torques developed in the driving motor tend to propagate motion of the drive system. This motion may be uniform if the linear velocity (in the case of translational motion) or the angular velocity (in the case of rotational motion) is constant, or non-uniform, as it occurs while starting, braking or changing the load on the drive.

In case of uniform motion the torque developed by the driving motor is to overcome any resisting torque offered by the driven equipment as well as the torque due to friction. In other words, only static resisting torques, commonly called as load torques, are to be counterbalanced, if the motion were uniform.

2.1 Types of Loads

Loads can be of two types—those which provide active torques and those which provide passive torques.

Active torques are due to either gravitational force or deformation in elastic bodies. The active torques due to gravitational pull are obtained in case of hoists, lifts or elevators and railway locomotives operating on gradients. Such torques are also developed during compression or release of springs. Since the functioning of hoisting mechanisms, operation of locomotives on gradients and compression or release of springs are all associated with a change in potential energy of the drive, active torques are also closely connected to the potential energy. When a load is moved upwards or a spring is compressed, the stored potential energy increases and the active torque developed opposes the action that takes place, i.e., the torque is directed against the upward movement or compression. On the other hand, when a load is brought downwards or a spring is released the stored potential energy decreases and torque associated with it aids the action. Thus, it can be seen that the active torques continue to act in the same direction even after the direction of the drive has been reversed.

Passive torques are those due to friction or due to shear and deformation in inelastic bodies (lathes, fans, pumps etc.). They always oppose motion, retarding the rotation of the driven machine. Moreover, with change in direction of motion, the sense of torque also changes. For example, when a weight is being lifted up, the friction torque adds to the useful torque, but when lowered down it subtracts from the latter.

2.2 Quadrantal Diagram of Speed-Torque Characteristics

In view of the fact that both active and passive load torques can be present in general, in a drive system, the motor driving the load may operate in different regimes—not only as a motor, but for specific periods, also as a generator and as a brake. Further, in many applications, the motor may be required to run in both directions. Therefore, in sketching the speed torque characteristics of either the load or the motor, it is preferable to use all four quadrants of the speed-torque plane for plotting, rather than to confine the characteristics to the first quadrant alone. When drawn in this manner, the diagram is referred to as quadrantal digaram.

The conventions used for positive and negative values of speed, motor torque and load torque in a diagram of this type must be understood very clearly. The speed is assumed to have a positive sign, if the direction of rotation is anticlockwise or is in a such a way to cause an 'upward' or 'forward' motion of the drive. In the case of reversible drives, the positive sign for speed may have to be assigned arbitrarily either to anticlockwise or clockwise direction of rotation.

The motor torque is said to be positive if it produces an increase in speed in the positive sense. The load torque is assigned a positive sign when it is directed against the motor torque.

Figure 2.1 shows the four quadrant operation of a motor driving a hoist consisting of a cage with or without load, a rope wound onto a drum to hoist the cage and a balance weight of magnitude greater than that of the empty cage but less than that of the loaded one. The arrows in this figure indicate the actual directions of motor torque, load torque and motion in the four quadrants. It can be easily seen that they correspond to the sign conventions stated earlier for speed, motor torque and load torque.

The load torque of the hoisting mechanism may be assumed to be constant, i.e., independent of speed, since the forces due to friction and windage are small in the case of low speed hoists and the torque is primarily due to the gravitational pull on the cage. This torque being an active load torque doesn't change its sign even when the direction of rotation of the driving motor is reversed. Therefore, the speed torque curves of a hoist load can be represented by means of vertical lines passing through two quadrants. The speed torque characteristic of a loaded hoist is shown in Fig. 2.1 by means of the vertical line passing the first and fourth quadrants. Since the counterweight is assumed to be heavier than the empty cage, the inherent tendency of the load, viz., the empty cage is to move in an opposite direction to that of load presented by the loaded cage and hence the speed torque curve of the unloaded hoist is represented by the vertical line passing through second and third quadrants.

In the first quadrant the load torque acts in a direction opposite to that

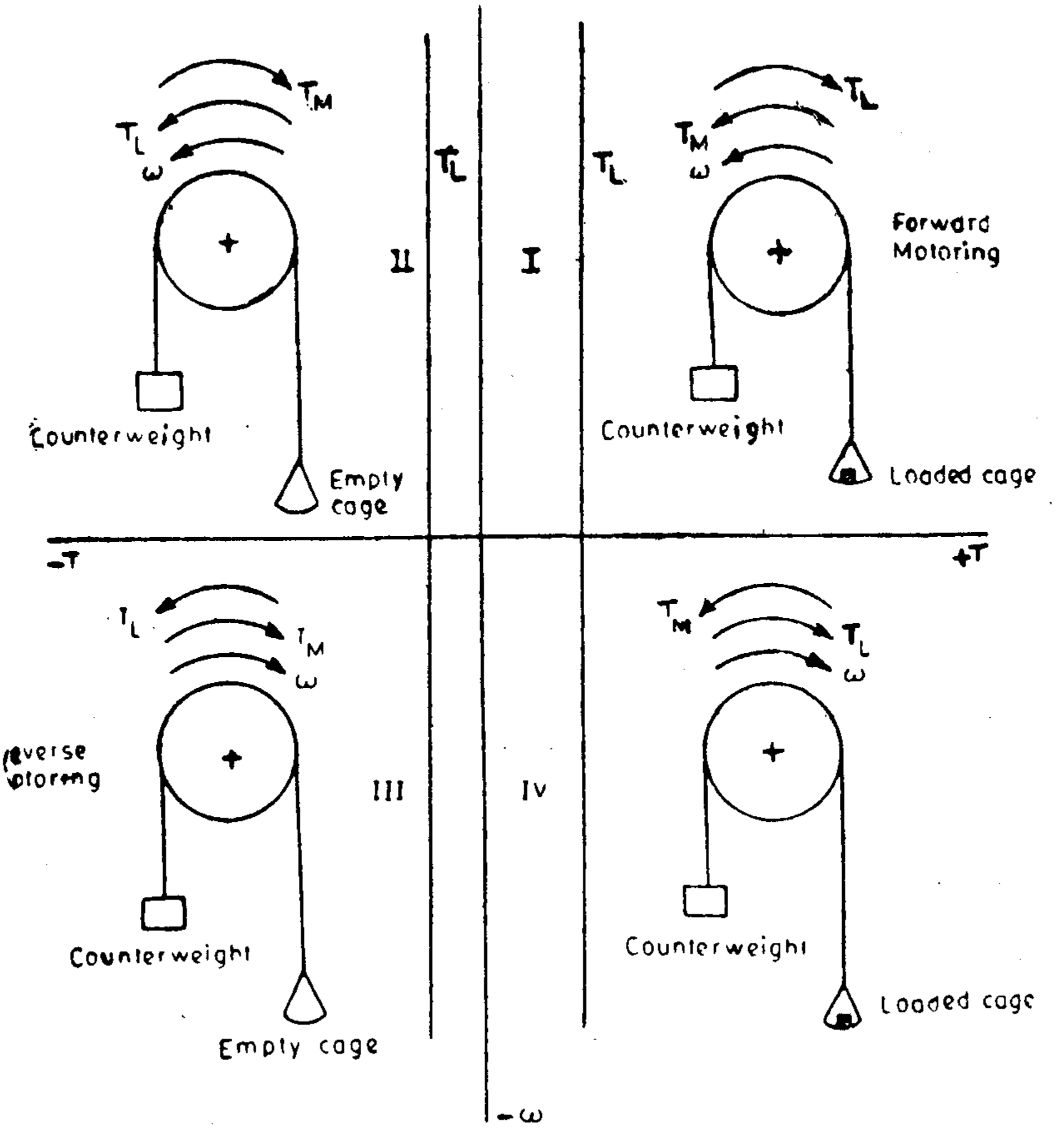

Fig. 2.1. Four quadrant operation of a motor driving a hoist load.

of rotation. Hence, to drive the loaded hoist up, the developed torque in the motor T_M must act in the same direction as the speed of rotation, i.e., T_M should be of positive sign. Since the speed is also of positive sign, being an upward motion, the power will also have a positive sign, i.e., the drive is said to be 'motoring'. Quadrant I is arbitrarily and conventionally, thus, designated as 'forward motoring quadrant'.

The hoisting up of the unloaded cage is represented in the second quadrant. Since the counterweight is heavier than the empty cage, the speed at which the hoist is moved upwards may reach a dangerously high value. In order to avoid this, the motor torque must act opposite to the direction of rotation, i.e., the motor should switch over to a braking or generator regime. Note that T_M will have a negative sign and speed still has a positive sign, giving power a negative sign, corresponding to the generator or braking operation.

The third quadrant represents the downward motion of the empty cage. The downward journey of the cage is prevented by the torque due to the counterweight and friction at the transmitting parts. Therefore, in order to move the cage downwards, the motor torque must act in the same direction as the motion of the cage. The electrical machine acts as a motor as in the first quadrant, but in the reverse direction. Thus quadrant III becomes 'reverse motoring'. The motor torque has a negative sign (because it causes an increase in speed in the negative sense) and the speed also has a negative sign (being a downward motion). Power, thus, has a positive sign.

The downward motion of the loaded cage is shown in the fourth quadrant. The motion can take place under the action of load itself, without the use of any motor. But, the speed of downward motion can be dangerously high. Therefore, in this case, the electrical machine must act as a brake limiting the speed of the downward motion of the hoist. The motor torque has a positive sign since it causes a decrease in speed in the downward motion. The speed, of course, has a negative sign, being a downward journey. The power, thus, acquires a negative sign, corresponding to the braking operation of the motor.

A second basic type of loading that occurs is the one characterized by dry friction. This type of load presents to the motor a passive torque, which is essentially independent of speed. It is characterized also by the requirement of an extra torque at very near zero speed. In power applications it is, often, called as the break away torque and in control systems, it is referred to as stiction (derived from sticking friction). The speed-torque curves for this type of load are shown in Fig. 2.2.

Another type of friction loading is used by control system engineers and is known as viscous friction. It is a force or torque loading whose magnitude is directly proportional to the speed. The viscous friction torque speed curves are illustrated in Fig. 2.3. Calendering machines, Eddy current brakes

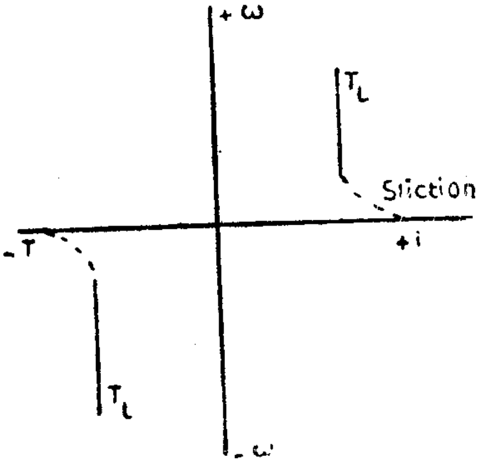

Fig. 2.2. Speed torque curve of dry friction load.

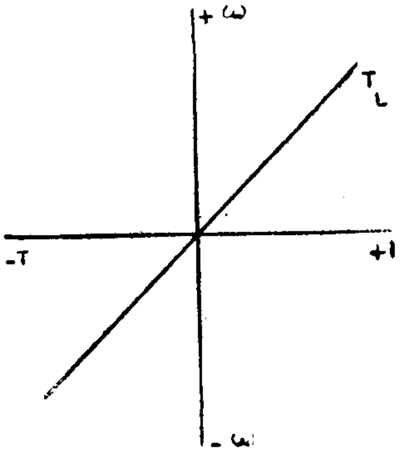

Fig. 2.3. Speed torque curves of viscous friction load.

and separately excited dc generators feeding fixed resistance loads have such speed-torque characteristics.

Yet another basic type of load torque is one whose magnitude is proportional to some power of the speed. Such a load is best illustrated by a fan or blower. The torque produced by the fan is directly proportional to the square of the speed throughout the range of usable fan speeds. The speed-torque curves for the fan type of load are presented in Fig. 2.4. Centrifugal pumps, propellers in ships or aeroplanes also have the same type of speed-torque characteristic.

Hyperbolic speed-torque characteristic (load torque being inversely proportional to speed or load power remaining constant), as shown in Fig. 2.5, is associated with certain type of lathes, boring machines, milling machines, steel mill coilers, etc.

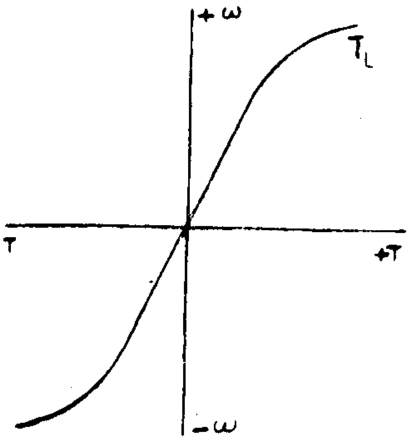

Fig. 2.4. Speed torque curve of a fan type load.

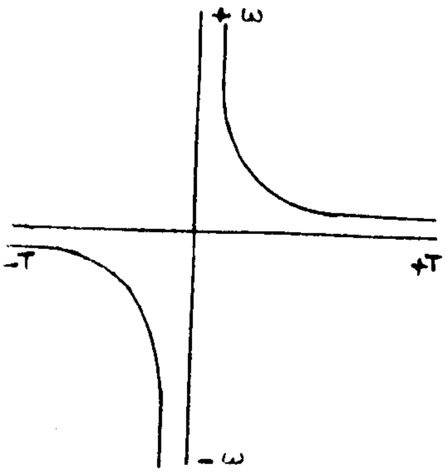

Fig. 2.5. Speed torque curve of a constant power load.

In general, the load torque in any specified application may consist of any of the above mentioned loads in varying proportions.

2.3 Load Torques that Depend on the Path or Position Taken by the Load During Motion

In the preceding section, we have been considering load torques which vary as a function of speed. However, load torques, that depend not only on speed but also on the nature of the path traced out by the load during its motion, are present both in hoisting mechanisms and transport systems. For instance, the resistance to motion of a train travelling upgradient or taking a turn depends on the magnitude of the gradient or the radius of curvature of the track respectively.

The force resisting the motion of a train travelling upgradient, as shown in Fig. 2.6 is given by

$$F_G = W \sin \alpha \cong W \tan \alpha \text{ } (\alpha\text{, being usually small})$$

$$= W \frac{G}{1000} \text{ kg,} \qquad (2.1)$$

where W = dead weight of the train or any other transport system, in kg, and G = gradient expressed as a rise in metres in a track distance of 1000 metres.

The tractive force required to overcome curve resistance is given by the empirical formula stated below:

$$F_c = \frac{700}{R} W \text{ kg}, \tag{2.2}$$

where R is the radius of curvature in metres.

In hoisting mechanisms in which tail ropes or balancing ropes are not used (Fig. 2.7) the load torque is not only due to the weight of the unloaded or the loaded cage but also due to that of the lifting ropes or cables.

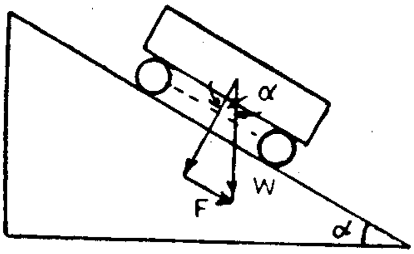

Fig. 2.6. Forces during the upgradient motion of a train.

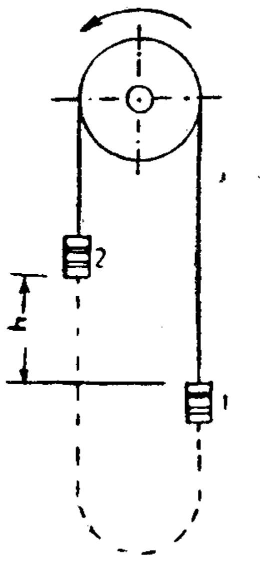

Fig. 2.7. Hoisting mechanism.

The latter depends on the position of the two cages. When cage 1 is at the bottom most position and is to be lifted upwards, the entire weight of the rope is also to be moved up. When both cages remain at the same height, the weight of the rope to be lifted up becomes zero, since the weight of the ropes on both sides balance each other. When cage 1 is at a higher position than cage 2, a portion of the weight of the rope acts in such a way as to aid the upward motion of cage 1. In fact, when cage 1 occupies the topmost position, the whole weight of the rope aids the upward movement.

The force that resists the upward motion of the load F_r due to the varying weight of the rope depending on the position of the load, is given as:

$$F_r = W_r \left(1 - \frac{2x}{h}\right) \text{ kg}, \tag{2.3}$$

where W_r = total weight of the rope, in kg,

x = height of the cage at any arbitrary position from the bottom most position in m, and

h = the desired maximum height to which the cage is to be moved upwards, in m.

Since, for very high values of 'h', the weight of the rope may be considerably greater than that of the load to be lifted upwards, the force F_r affects, to a large extent, the performance of the drive used in hoisting mechanisms. By using tail ropes, as shown by means of dotted lines in Fig. 2.7, the weight of the connecting rope can be balanced and more or less smooth movement of the cages can be ensured.

Another example of a load torque, which depends on path traced out during motion, is that of a planing machine. At a particular position of the moving table containing the workpiece, the load torque comes in the form of a sudden blow; in a different position, after the cutter has come out of the job, the magnitude of the load torque decreases sharply.

2.4 Load Torques That Vary with Angle of Displacement of the Shaft

In all machines, having crankshafts, for example, in reciprocating pumps and compressors, frame saws, weaving looms, rocking pumps used in petroleum industry etc., load torque is a function of the position of the crank, i.e., the angular displacement of the shaft or rotor of the motor. Load torque in drives used for steering ships also belongs to this category.

Figure 2.8 shows the approximate relationship between the load torque and angular displacement of the shaft 'θ' for a reciprocating compressor. It is of the form $T_L = f(\theta)$, where θ itself varies with time. For all such machines, the load torque T_L can be resolved into two components—one of constant magnitude T_{av} and the other a variable T'_L, which changes periodically in magnitude depending on the angular position of the shaft. Such load torque characteristics, can, for simplicity, be represented by

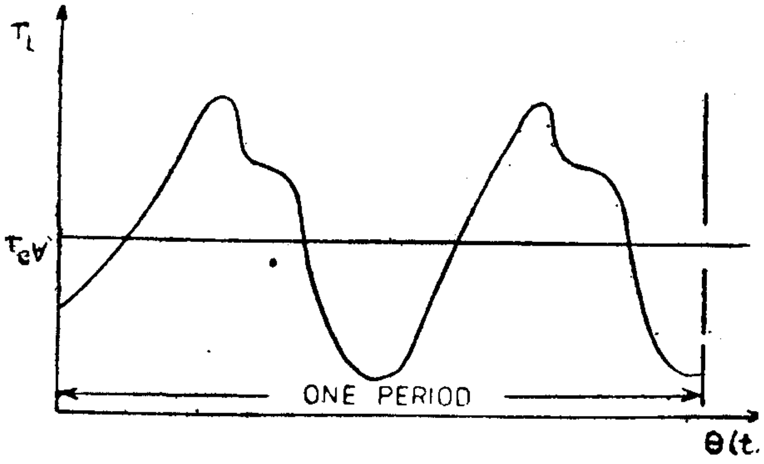

Fig. 2.8. Speed-torque characteristic of a reciprocating compressor.

Fourier series as a sum of oscillations of fundamental and harmonic frequencies, i.e.,

$$T'_L = \sum_{r=0}^{m} T'_{Lr} \sin(r\theta + \phi_r) \tag{2.4}$$

$\theta = \omega t$, where ω represents the angular speed of the shaft of the motor driving the compressor.

During changes in speed, since only small deviations from a fixed value of speed ω_a occur, the angular displacement can be represented by $\theta = (\omega_a + \Delta\omega)\, t$. Then, the variable portion of the load torque may be expressed as

$$T'_L = \sum_{0}^{m} T'_{Lr} \sin[(r\omega_a t + \phi_r) + r.\Delta\omega t] \tag{2.5}$$

The term $r.\Delta\omega t$ being of very small magnitude can be neglected. Thus, restricting to small deviations in angle from the equilibrium position, a load torque which varies with the angular displacement of the shaft can be transformed to one which varies periodically with respect to time.

2.5 Load Torques That Vary with Time

Of equal or perhaps greater importance in motor selection is the variation of load torque with time. This variation, in certain applications, can be periodic and repetitive, one cycle of variation being called a duty cycle. It is convenient to classify different types of loads under the following groups:

(a) *Continuous, constant loads*: Centrifugal pumps or fans operating for a long time under the same conditions; paper-making machines etc.

(b) *Continuous, variable loads*: Metal cutting lathes; hoisting winches; conveyors etc.

(c) *Pulsating loads*: Reciprocating pumps and compressors; frame saws, textile looms and, generally, all machines having crank shaft.

(d) *Impact loads*: Apparent, regular and repetitive load peaks or pulses which occur in rolling mills, presses, shearing machines, forging hammers etc. Drives for such machines are characterized by heavy flywheels.

(e) *Short time intermittent loads*: Almost all forms of cranes and hoisting mechanisms; excavators; roll trains etc.

(f) *Short time loads*: Motor-generotor sets for charging batteries; servomotors used for remote control of clamping rods of drilling machines.

Certain machines like stone crushers and ball mills do not strictly fall under any of the above groups. If these loads were characterized by frequent impacts of comparatively small peaks, it would be more appropriate to classify them under continuous variable loads rather than under impact loads. Sometimes, it is difficult to distinguish pulsating loads from impact loads, since both of them are periodic in nature and, hence, may be expressed as a sum of sinusoidal waves of different amplitude, frequency and phase.

One and the same machine can be represented by a load torque which varies either with speed or with time. For example, a fan load whose load torque is proportional to the square of the speed, is also a continuous,

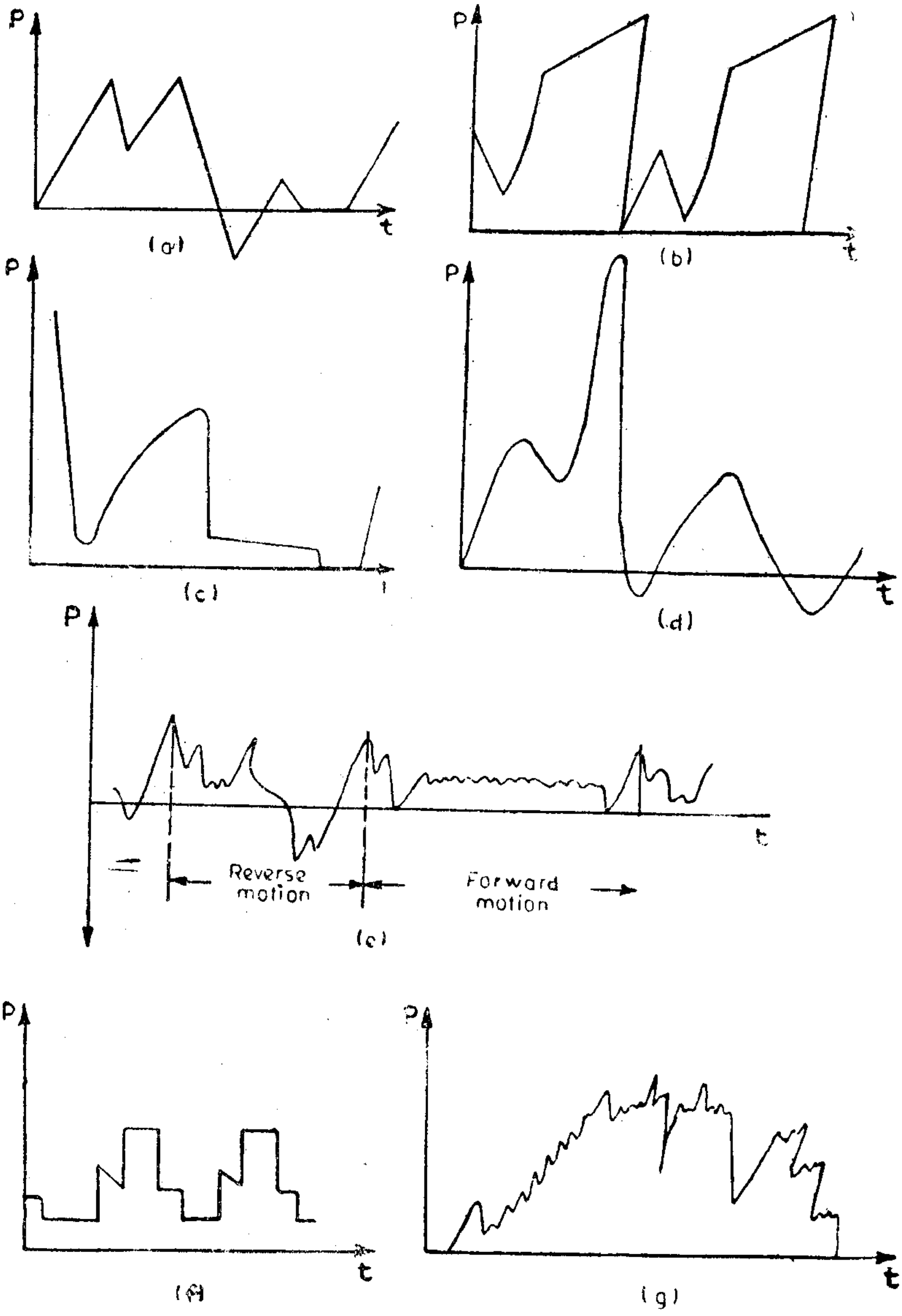

Fig. 2.9. Power-time curves of some common loads:
(a) Mine hoist
(b) Polishing machine
(c) Shearing machine for cutting steel
(d) Textile loom
(e) Planing machine
(f) Drilling machine
(g) Grinding machine.

constant load. Load torque of a crane is independent of speed and also short time intermittent in nature. Rocking pumps for petroleum have a load which vary with angular position of the shaft, but can also be classified as a pulsating load.

The nature of load (power) variation with respect to time corresponding to certain common applications is shown in Fig. 2.9.

2.6 Dynamics of Motor-Load Combination

The motor and the load that it drives can be represented by the rotational system shown in Fig. 2.10. Although the load, in general, may not rotate at the same speed as the motor, it is convenient to represent it in this manner so that all parts of the motor-load system have the same angular velocity. In case, the speed of the load differs from that of the motor, one can find out an equivalent system (as explained later).

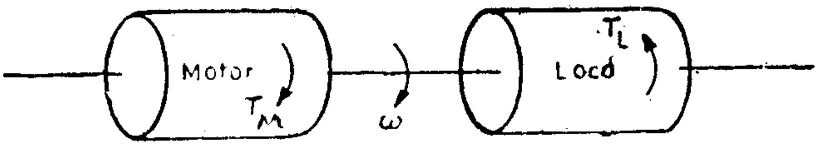

Fig. 2.10. Motor-load system.

The basic torque equation, known as the equation of motion, for the above motor-load system, is written as

$$T_M = T_L + J\frac{d\omega}{dt} \tag{2.6}$$

where T_M and T_L denote motor and load torque measured in N-m; J, the moment of inertia of drive system in kg-m^2 and ω, the angular velocity in mechanical radians/sec.

In the above equation the motor torque is considered as an applied torque and the load torque as a resisting torque.

From the above equation, it is possible to determine the different states at which an electric drive causing rotational motion can remain.

(i) $T_M > T_L$, i.e., $d\omega/dt > 0$, i.e., the drive will be accelerating, in particular, picking up speed to reach rated speed.

(ii) $T_M < T_L$, i.e., $d\omega/dt < 0$, i.e., the drive will be decelerating and, particularly, coming to rest.

(iii) $T_M = T_L$, i.e., $d\omega/dt = 0$, i.e., the motor will continue to run at the same speed, if it were running or will continue to be at rest, if it were not running.

The above statements, namely, that when $T_M > T_L$ the drive accelerates and that when $T_M < T_L$ the drive decelerates, are valid only when T_L happens to be a passive load. The reverse may occur with active loads. For example, if we were to switch on the motor for hoisting up a winch, while it is coming down on its own weight, until the direction of rotation changes, deceleration of the drive and not acceleration takes place, when $T_M > T_L$. In case $T_M < T_L$ in the above situation when the motor has been switched on for moving the winch up, the load will continue to come down and the motor will accelerate instead of decelerating.

The term $J\, d\omega/dt$ which represents the inertia torque, is also known as dynamic torque, since it is present only during transient conditions, i.e., when the speed of the drive varies. During acceleration of the drive, the inertia torque is directed against motion, but during braking it maintains the motion of the drive. Thus, inertia torque is determined both in magnitude and sign, as the algebraic sum of the motor and load torques.

In view of the above, the signs for T_M and T_L in Eqn. (2.6) correspond to motoring operation of the driving machine and to passive load torque or to a braking torque caused by active loads, respectively. The equation of motion can, in general, be written as:

$$\pm T_M = \pm T_L + \frac{J\, d\omega}{dt} \tag{2.7}$$

The signs to be associated with T_M and T_L in Eqn. (2.7) depend, as indicated earlier, on the regime of operation of the driving motor and the nature of load torque. The equation of motion enables us to determine the variation of torque, current and speed with respect to time, during transient operation of the drive.

2.6.1 Equivalent System

Seldom is a motor shaft directly coupled to load shafts. In general, the different loads connected to the motor will have different speed requirements. Speed changing mechanisms such as gears, V-belts, etc., will be used to obtain different speeds. Since the ultimate objective is to select a motor suitable for the application, it is desirable to refer all mechanical quantities such as load torque, inertia torque, etc., to one single axis of rotation, conveniently, the output shaft of the motor. The principle of conservation of energy will be used for this purpose.

2.6.2 Determination of Referred Load Torque

Let the speed of the motor shaft be ω_M and that of the load be ω_L.

Equating power, we have

$$T_L \cdot \omega_L \frac{1}{\eta} = T'_L\, \omega_M$$

i.e.,

$$T'_L = T_L \cdot \frac{\omega_L}{\omega_M} \times \frac{1}{\eta} = \frac{T_L}{i\eta}, \tag{2.8}$$

where T_L = load torque

T'_L = load torque referred to the motor shaft,

$i = \frac{\omega_M}{\omega_L}$ = speed transmission ratio (gear ratio), and

η = efficiency of transmission.