

Autonomous Underwater Vehicles

**Modeling, Control Design,
and Simulation**



Sabiha Wadoo • Pushkin Kachroo



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Dedication

To my children Saami and Shireen Sabiha Wadoo

To my children Axenya and Sheen Pushkin Kachroo

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Preface

Control design of autonomous underwater vehicles is an important area for researchers in the control systems community. Control is generally difficult to achieve due to nonlinear dynamics, uncertain models, and the presence of disturbances that are hard to measure or estimate. This book presents a new approach to the modeling and control design of autonomous underwater vehicles.

Kinematic and dynamic nonlinear models for autonomous underwater vehicles are discussed in this book. Controllability of autonomous underwater vehicles for different motion planning tasks is reviewed. The book combines the feedback control design for both kinematic and dynamic models. The feedback control is also achieved in the presence of uncertainties, thereby making the control design robust.

This work is intended for students and researchers, as there is more development that is needed in this particular area. The results of this book can be extended to obtain advanced control strategies and design schemes for autonomous underwater vehicles and other similar problems falling in the area of nonlinear control.

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1 Introduction

1.1 OVERVIEW

The work presented in this book is concerned with developing models that can be used for the motion planning and feedback control design of underwater vehicles. Underwater vehicles present difficult control system design problems due to their nonlinear dynamics, uncertain models, and the presence of disturbances that are difficult to measure or estimate. Many problems must be solved to make robotic underwater vehicles a reality. The dynamic control of the vehicle needs to guarantee stability and perform consistently. The dynamics of autonomous underwater vehicles present a difficult control system design problem that traditional linear design methodologies cannot accommodate easily. The dynamics are fundamentally nonlinear in nature. Hydrodynamic coefficients often are poorly known, and a variety of immeasurable disturbances are present due to currents.

In this book, a new approach of feedback control methodology is developed to the accurate trajectory control and point stabilization of underwater vehicles. The feedback control is developed in both the absence and presence of uncertainties. The methods can deal with nonlinear dynamics directly, and are shown to be stable in the presence of disturbances, thereby making the control robust.

The first part of the book presents the study concerning the applicability of kinematic-based control of underwater vehicles. The methods of motion planning and feedback control for a kinematic model of an underwater vehicle are developed. The kinematic model of an underwater vehicle falls into a general category of nonholonomic systems. The system is characterized by nonholonomic constraints on its generalized velocities. The motion planning problem for this system, with constraints on the velocities, is transformed into a control problem having fewer control inputs than the degrees of freedom. The nonlinear controllability issues for the system are also studied. For the design of feedback controllers, the system is transformed into chained and power forms. The methods of transforming the kinematic model of the system into these forms are discussed. Differential geometric control theory and nonlinear system analysis and control design techniques, and the results of recent research and study in the motion planning of nonholonomic systems are used and presented for the support of the work.

The kinematic model of the autonomous underwater vehicle is developed, and the feedback controller design for the same is presented in detail and simulation results are obtained. Also, a brief mathematical analysis of the concepts involved in the study of controllability, control design, and modeling is presented.

The kinematic-based control addresses the motion planning for a kinematic model of an underwater vehicle. The kinematic model belongs to nonholonomic systems. The control model for such systems is drift-free, nonlinear, and underactuated, given by

$$\dot{q} = g_1(q)v_1 + g_2(q)v_2 + \cdots + g_m(q)v_m \quad (1.1)$$

Here $q \in M$ is the state vector of the system, M is the state space, and $M \subset R^n$, where n is the dimension of the configuration space M , to which the vector q belongs. Vector $v \in R^m$ is the input or the control vector of dimension m . Vectors $g_i(q) \in R^n; i = 1, 2, \dots, m$ are vector fields on M and are assumed to be smooth and linear time invariant. The system is called drift-free because the system state does not change under zero-input conditions. Also, the system is underactuated because the dimension of the space spanned by the control vector is less than the dimension of the configuration space.

A special case of Equation 1.1 with two inputs was presented in [1]. In [1] the motion planning tasks for a car-like robot were defined and the feedback control design was studied. The control was achieved using various control strategies for each task. The kinematic-based control of an underwater vehicle is an extension of the work presented in [1]. The extended problem is higher dimensional with four inputs. In this book the controllability of the system is discussed and proved as related to motion planning. We present feedback control laws that give global stabilization of the vehicle about a desired trajectory and about a point. This is achieved by transforming the kinematic into canonical chained and power forms. The book presents the method of converting the kinematic model into chained and power forms via state feedback and coordinate transformation.

For trajectory tracking of underwater vehicles [23] proposed a stable tracking control method based on a Lyapunov function. In [22] and [23] a Lyapunov-like function is used to develop a nonlinear feedback control scheme. The control achieves global stabilization about a desired trajectory. However, the system is not point stabilizable with the use of the proposed controller. In this book we will be making use of the full state feedback (approximate linearization) scheme for trajectory tracking. This scheme results in local asymptotic stabilization only. Exact nonlinear control (full state linearization) design is used to achieve the global stabilization. In this case static state feedback fails to achieve the goal. However, the dynamic state feedback is used to serve the purpose. Here the control design is done using the chained form system.

The kinematic model of an underwater vehicle belongs to a class of systems that cannot be stabilized by a pure state feedback law [2]. The design of globally asymptotic stabilizing controllers for nonholonomic systems is challenging. The design is difficult in a sense that no time-invariant smooth static stabilizing controller exists for such systems [2]. Various control schemes have been adopted for this purpose.

One way to deal with this is to use time-varying, smooth controllers. This approach has been extensively studied in [3] and [4]. In [3] it is shown that time-varying smooth control laws for driftless systems have necessarily algebraic (not exponential) convergence rates. Asymptotic stabilization for underwater vehicles using time-varying smooth feedback laws was achieved in [4].

Another alternative is the use of the nonsmooth feedback controllers that can achieve exponential convergence. These schemes have been proposed in [5] and [6]. In [25] a discontinuous piecewise smooth control law was proposed and exponential convergence to a constant desired configuration was achieved. In [15] a nonsmooth time-invariant controller was proposed to achieve the exponential convergence with stability to a constant desired configuration. The controller was implemented using

the chained form. In this book, we have adopted the former approach, that is, we used a time-varying and smooth feedback. The control design method for stabilization used herein is adopted from [7]. To this end, a transformation of the kinematic model into power form is derived. The controller achieves global stabilization to a constant configuration for an underwater vehicle.

The latter part of the book addresses the feedback control and robust control design for a dynamic model of an underwater vehicle. The motion of underwater vehicles is presented as a kinematic and a dynamic model. The three-dimensional vehicle motion may be described in terms of the twelve nonlinear system equations:

$$\begin{aligned} M \frac{dv(t)}{dt} &= f(v, q) + g(v, q)u(t) \\ \frac{dq(t)}{dt} &= h(v, q) \end{aligned} \tag{1.2}$$

The first equation is the dynamic model of the system, and the second is the kinematic model. The vector $v(t)$ is the velocity vector and $q(t)$ is the position vector in the fixed frame coordinates. In order to design the feedback control for point stabilization of the dynamic model, the control methodology is nonlinear feedback linearization. The design technique adopted for the dynamic control is *backstepping*. The book also discusses the feedback control for these models in the presence of uncertainties where the goal is to design the controllers to minimize the effect of uncertainties. The control is achieved using Lyapunov redesign and *robust backstepping* for uncertain models. In both methods we achieve the control design for the kinematic model first, which is then used to achieve the overall control design of the dynamic model. The outline of the chapters is summarized below.

Chapter 2 gives an introduction and general overview of the motion planning of autonomous vehicles. The concepts of nonholonomy, underactuated systems, and a kinematic model of the nonholonomic systems, and some examples are shown. Then the general problem of motion planning and the related issues are formulated for a class of the nonholonomic systems, with a review of some particular applications.

Chapter 3 presents an overview and detailed analysis of the related motion planning tasks of an autonomous underwater vehicle. The chapter presents in detail the derivation of the mathematical modeling of the system. For motion planning tasks, the kinematic model of the system is obtained and the issues related to nonlinear controllability of the system are studied in detail. Finally, for the purpose of control design, the system is converted into a chained form. The method of converting a multi-input nonholonomic system into a chained form is also discussed.

Chapter 4 presents the control design and the simulation results obtained for the model of an underwater vehicle developed in Chapter 3. The feedback control design is developed using the kinematic model of the system. The performance of the controllers using various techniques of control design is obtained and evaluated for different motion planning tasks, such as trajectory tracking, point stabilization, and path following. The chapter also presents the simulation results obtained for

different controllers. The simulation results are used to compare and evaluate the performance of the various controllers for different paths following tasks.

Chapter 5 presents the control design of the dynamic model of an underwater vehicle. The control design is obtained and evaluated for a motion planning task of point stabilization. The control design technique adopted here is feedback linearization using backstepping for a dynamic model. The problem of control and stability is formulated and sufficient conditions for Lyapunov stability for the designed controllers are derived.

Chapter 6 presents a design of robust nonlinear feedback controllers for the dynamic model representing underwater vehicles. We discuss the feedback control for these models in the presence of uncertainties where the goal is to design the controllers to minimize the effect of uncertainties on the performance of the system. The robust controllers are designed for point stabilization using Lyapunov redesign and robust backstepping methods.

1.2 EXAMPLES OF UNDERWATER VEHICLE CONSTRUCTION

This section provides a brief overview of the construction of underwater vehicles. Readers are encouraged to study [33] and [34] for more details.

Underwater vehicles have two main issues to deal with. They need to be able to stay underwater and to maneuver. The vehicles can be designed so that they stay underwater, so that their body weight is water neutral, or they can use actuation to actively stay underwater. They also need actuation to be able to go underwater from the water surface, move around underwater, and also come back up to the surface.

Submarines can be dynamic or static. Dynamic submarines usually will float on water and are actively pushed down by propellers to keep them underwater. Static submarines, on the other hand, add water into a submarine chamber (called a ballast tank) to add weight to it, so the submarine can get heavier. When it wants to come up, it uses compressed gas to push the water out, so that the submarine becomes light again.

Remote-controlled (RC) submarines, which can be purchased in many stores, have a motor that drives a propeller at the back that pushes the vehicle forward, and rudder and stern (and sail) planes to enable the submarines to produce yaw motion and pitch motion. Rudders enable the yaw motion by their rotation, and stern planes produce the pitch motion just like elevators do in RC planes. These actuators are shown in [Figure 1.1](#). A more detailed view of the rudder and stern is shown in [Figure 1.2](#).

The weight distribution keeps the submarine facing right side up by keeping the heavier weight at the bottom, not the top. The sail plane with the stern plane keeps the submarine balanced. The electronics is kept inside the water-tight container (WTC) and contains two servos and a DC motor. One servo controls the rudder angle and the other controls the stern. The location of the WTC and the weight distribution are shown in [Figure 1.3](#).

The WTC generally has three compartments. The first compartment will have the battery and speed controller, the second will have a ballast system, and the third the servos. Typically, the ballast system will have a tank with pressurized air and valves. The valves are used to allow water to flood in, to increase the weight of the submarine, therefore enabling it to go down, or to let water come out when the pressurized air is let into the chamber, so that the weight of the submarine is reduced, to enable it to rise. The

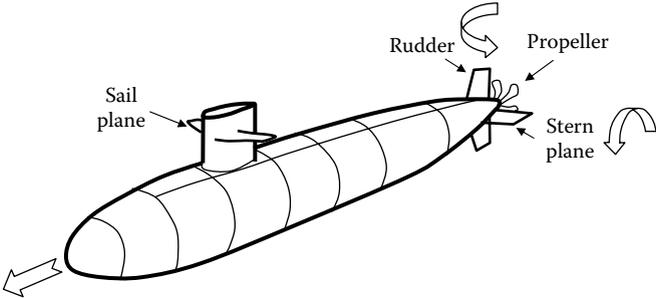


FIGURE 1.1 Submarine actuators.

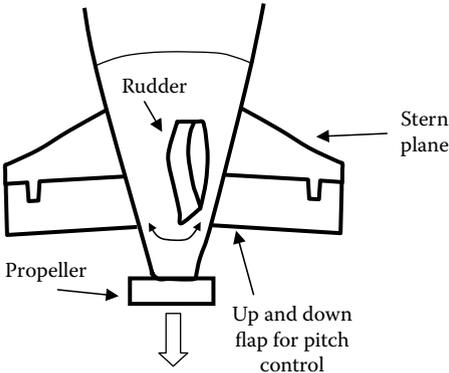


FIGURE 1.2 Submarine rudder and stern.

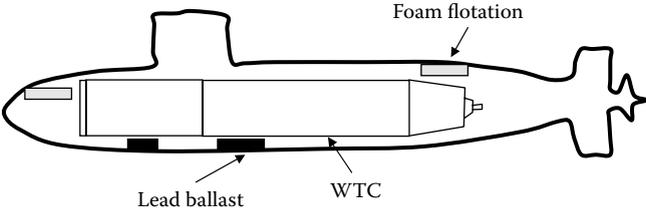


FIGURE 1.3 WTC location and weight distribution.

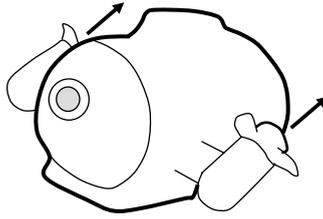


FIGURE 1.4 Twin-propeller-based dynamic RC submarine.

other electronic components that are usually present are the speed controller connected to the radio signal receiver, which receives the signal from the remote control to control the motor and servos; the pitch control using an accelerometer to control the pitch angle; and a microsafety, a device that turns the ballast system off if the radio signal is lost for some time, so that the submarine will float on top in case of a lost radio signal.

Submarines communicate underwater with sonar signals using audio frequencies. Submarines also use GPS for navigation, but that signal is lost underwater; therefore, an underwater inertial navigation system (INS) is used that is composed of accelerometers and gyroscopes to measure how the submarine is moving so that it can keep track of its position and orientation.

Dynamic submarines don't have the ballast system, and therefore they have to be actively forced down in the water; otherwise, they float up on the surface. Many inexpensive dynamic RC submarines are built using twin motors that are used to dive, surface, turn, etc. The angle of the propellers can be changed before putting the submarine into water to get different movements. One such submarine is illustrated in Figure 1.4.

1.2.1 PROPELLER PRINCIPLE

Since underwater vehicles have propellers, this section will provide a brief description of the theory of propeller action. Propeller thrust is produced due to the forces that are created by the “wings” that are part of the propellers. We present the fundamentals of how force is produced in the wings, and then how propellers produce thrust because of them.

1.2.1.1 Wings

To understand how the air flowing around a wing section produces lift, let us study Figure 1.5. The figure shows fluid flowing around a wing section. The streamline (a line

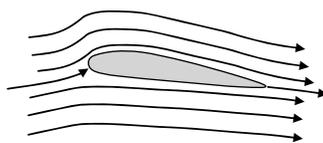


FIGURE 1.5 Airflow around a wing section (Kutta condition).

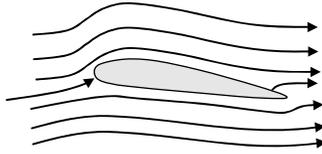


FIGURE 1.6 Noncirculatory airflow around a wing section.

whose tangent shows the direction of flow at every point) on the left that ends with an arrow on the wing surface is the one that reaches the stagnation point on the wing. This means that above that the fluid flows on top of the wing, and below that point the fluid flows under the wing. The figure shows that the fluid from the top and bottom will have the same magnitude and direction at the trailing edge. This is a condition (Kutta condition [33]) that creates lift from fluid flowing around a wing at some angle of attack. If we place a wing in a fluid flow, we can expect the streamlines to be as shown in Figure 1.6. This fluid flow, however, has no circulation. Without fluid circulation, there can be no lift. In fact, the lift on a wing is proportional to the air circulation about the wing (Kutta–Joukowski theorem [34]).

According to the Kutta–Joukowski theorem, the lift per unit length of a wing is given by

$$\text{Lift} = \text{airspeed} \times \text{air circulation} \times \text{air density}$$

The fluid flow that follows the Kutta condition (the trailing edge boundary condition) can be obtained by superimposing a circulatory airflow on the noncirculatory fluid flow. This is shown in Figure 1.7.

There is another theorem (Kelvin’s circulation theorem [33]) that says that the rate of change of circulation around a closed curve that consists of the same fluid material is zero. When a vehicle such as an airplane starts accelerating, it does not follow the Kutta condition, and consequently, it does not have circulation. After the speed increases further, the Kutta condition is satisfied (creating circulation around the wings), and to make the total circulation zero, another circulating air is created behind the wing (called the *wake*). This is shown in Figure 1.8.

Due to the addition of clockwise circulation that is flowing in the direction of the regular airflow, the speed on top of the wing is higher than the fluid speed below the wing. Bernoulli’s theorem shows that where the airspeed is high, there the pressure is low, and where the airspeed is low, there the air pressure is high [34]. Since the air

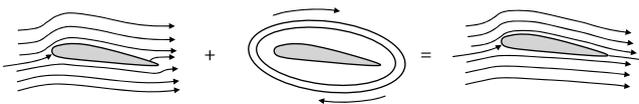


FIGURE 1.7 Total fluid flow around a wing section.