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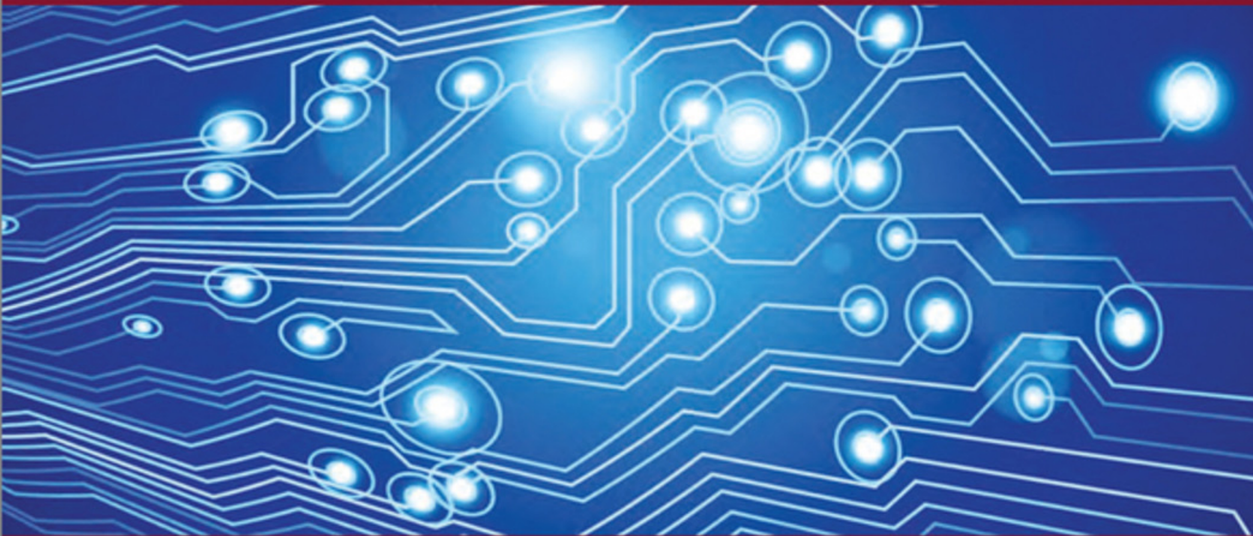

Control of Switching Systems by Invariance Analysis

Application to Power Electronics

Laurent Fribourg and Romain Soulat

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Contents

Preface

Switched systems are embedded devices widespread in industrial applications such as power electronics and automotive control. They consist of continuous-time dynamical subsystems and a rule that controls the switching between them. Under a suitable control rule, the system can improve its steady-state performance and meet essential properties, such as safety and stability, in desirable operating zones. We show in this book that such controller synthesis problems are related to the construction of appropriate invariants of the state space, which approximate the limit sets of the system trajectories. We present several approaches of invariant construction based on techniques of state space decomposition and backward/forward fixed-point computation, and perform them directly on the continuous state space, or indirectly on discrete abstractions. All these methods are illustrated in a number of case studies, mainly taken from the field of power electronics.

Acknowledgments

Most of this work was produced in the framework of projects BOOST and BOOST2, which are cooperations between Systems et Application des Technologies de l'Information et de l'Energie (SATIE) Electronics Laboratory and Laboratoire Specification et Verification (LSV) Computer Science Laboratory at ENS Cachan. We are grateful to Institut Farman, ENS Cachan, and Centre National de la Recherche Scientifique (CNRS) for supporting these interdisciplinary projects.

Bertrand Revol (SATIE) was the co-leader of projects BOOST and BOOST2 and initiated the cooperation with LSV by proposing concrete case studies of control of devices of power electronics. Chapter 5 is based on technical reports [FEL 12a, FEL 12b], which have been written in collaboration of Revol and his colleagues Gilles Feld, Denis Labrousse and Stéphane Lefebvre.

When coming to the design and implementation of the tool of decomposition (Appendix 5), Ulrich Kühne implemented an interface with PPL [PPL 13] which allows us to calculate a sequence of nested invariants, and visualize it as an animation.

Finally, we are most grateful to Étienne André for his very helpful comments on an earlier draft of the manuscript.

Introduction

In recent years, there has been an increasing interest in applying renewable energy in electricity generation and transportation. In particular, much effort has been devoted to the improvement of robust and flexible control techniques of power converters in order to increase reliability and safety of operation. Due to their practical feasibility to achieve a high performance as well as natural digital implementation in signal processors, *switched controllers* are the most common type of controller to have been applied to power converters (see [CER 09, LIB 03, SUN 05]). Systems equipped with switched controllers are constituted of two parts: first, a family of continuous subsystems or *modes*; second, a *switching signal* that controls the selection of these modes. The switching signal can be state dependent and/or time dependent.

With respect to classical systems, an interest of switching controllers arises from the existence of systems that cannot be asymptotically stabilized by a single continuous feedback control law [BRO 83]. However, with switched systems, the steady-state operating condition is typically a *periodic solution* or *limit cycle*, not an equilibrium point. The relevant stability notion is *asymptotic orbital stability* or *practical stability*, which studies the conditions under which the system state evolves within certain subsets of the state space [LAS 61]. The problem of stabilization of switched dynamical systems is thus much more difficult in general than in classical control theory. In particular, instability phenomena can occur even when all the modes, taken separately, are stable.

Although the general control theory of switched systems is very difficult, special cases of these problems arise frequently in restricted contexts associated with control design, and may be simpler to solve specifically. It is thus suggested in [LIB 99] to stay in close contact with particular applications of switched systems. This is indeed what happens to the authors of this book: as researchers in LSV Computer Science Laboratory, we have cooperated with researchers of SATIE Electronics Laboratory in the framework of an interdisciplinary project relevant to the control of practical examples of switched systems in the domain of power electronics. We have thus focused on switching signals that operate with a fixed switched period denoted by τ. These signals are very common because of their ease of implementation. Also a fixed-period operation avoids potentially troublesome harmonic side-effects that may arise with varying frequency operation (see [GEY 08]). There are two types of periodic switched controllers: *state-independent* controllers that cyclically apply the same sequence of modes that has been computed off-line and *state-dependent* controllers that select modes dynamically according to the regions of the states at the switching instants. Furthermore, the dynamics of each subsystem obeying Ohm's electrical laws, are governed by *affine* differential equations. These systems can thus be viewed as special cases of *hybrid systems* (see [HEN 96]) combining affine continuous dynamics and discrete transitions taking place at instants that are integer multiples of τ. Such a subclass has been recently studied by many researchers such as Antoine Girard, Giordano Pola and Paulo Tabuada (see, e.g., [TAB 09]). These systems are called "time-triggered sampled versions of switched systems". We call them here simply $\mathcal{S}^2$-systems (for sampled switched systems).

In classical control theory, we make use of Lyapunov theory in order to analyze and stabilize controlled systems. Roughly speaking, Lyapunov functions are energy functions characterizing the state of the system that decrease until they reach a 0 level, which corresponds to a level where the system is stable. These Lyapunov functions can be extended in the framework of switched systems under the form of so-called "multiple Lyapunov functions" or "common Lyapunov functions", and plays a central role in, for example, [TAB 09]. However, there is no general method for finding appropriate Lyapunov functions, and we have preferred in this book to avoid using them. Instead, the theoretical tool that we have mainly used is based on the notion of the "(controlled) invariant" [BLA 99]. Note, however, that the two concepts of invariants and Lyapunov functions are closely related, and we can show (at least in the classical context) that the level sets of Lyapunov functions correspond to the boundaries of invariant sets, and that the converse holds.

This book focuses on the restricted class of sampled switched systems, and on methods for controlling them using invariants. We will exploit the construction of invariants in order to synthesize two main classes of controllers: *safety* controllers and *stability* controllers. Safety controllers aim at protecting the system from undesirable states, while stability controllers aim at driving the system to a steady-state operating condition. To synthesize safety controllers, we describe *indirect* methods working on an abstract discrete level, and *direct* methods working on the continuous state-space level. These methods adopt a classical *backward* computation of the reachable states. To synthesize stability controllers, we describe a method of state-space *decomposition* that allows us to construct limit-cyclic trajectories by iterated forward computation of the reachable states.

The control strategies synthesized by this method have been numerically simulated, and also successfully experimented on physical prototypes built by SATIE Electronics Laboratory. The code has been written in Octave [OCT 13], and is available at the end of this book. We hope that this book that surveys methods of the literature and presents some recent enhancements together with the description of the implementation code will be interesting for students and engineers, and will encourage them to use, experiment, adapt and improve these procedures.

This book is structured as follows. In Chapter 1, we formally define the model of $\mathcal{S}^2$-systems, and explain in Chapter 3, how to synthesize safety controllers for $\mathcal{S}^2$-systems. In Chapter 4, we explain how to synthesize stability controllers for $\mathcal{S}^2$-systems using an original procedure of state-space decomposition. We show in Chapter 5 how to apply the procedure for controlling an important application of power electronics. In Chapter 6, how to extend the procedure in order to synthesize robust safety controllers and reachability controllers is explained, and suggestions for how to use it for sensitivity analysis are given. The main results with some perspectives are reviewed in Conclusions and Perspectives. Notes citing sources and related works are given at the end of each chapter.

This book tries as much as possible to avoid too theoretical results, rather focusing on the practical ideas of control synthesis for sampled switched systems using invariance analysis. In particular, proofs of the results are usually omitted. An exception is presented in Chapter 4, where we do detail an original procedure of state-space decomposition, motivated by and applied to the analysis of concrete examples of power electronics.

1

Control Theory: Basic Concepts

This chapter presents basic concepts of control theory, which will be used in the remaining book.

In section 1.1, we present the general *control/plant* model. In section 1.2, we explain why the introduction of digital sensors and actuators in systems has fundamentally modified the issue of controlled stability. Finally, we introduce the model of *switched* systems, and explains their advantages compared with general systems (section 1.2.3). We then explain in section 1.3 how the notion of *invariant sets* can be used for proving safety and stability properties of controlled systems.

1.1. Model of control systems

A control system is generally divided into a controlled part, called a *plant*, and a *controller*. The plant is generally described as a dynamic time-invariant, possibly uncertain, system governed by equations of the form:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), w(t)) & [1.1] \\ y(t) = g(x(t)), & [1.2] \end{cases}$$

where $x(t) \in \mathbb{R}^n$ is the *system state*, $u(t) \in \mathbb{R}^m$ is the *control input* $y(t) \in \mathbb{R}^p$ is the *output*, $w(t) \in \mathcal{W} \subset \mathbb{R}^q$ is a *disturbance* (or external input) and $\mathcal{W}$ is an assigned compact set. We will refer to $\mathbb{R}^n$ as the *state space* of the system. The general theory of control focuses on *feedback control*: the controller is fed with state signal $x(t)$ coming from the plant, and issues a control input $u(t)$ to the plant. A typical layout of a feedback control system is shown in Figure 1.1. Under general conditions (continuity for u and w, and Lipschitz property for f), the system admits a unique solution $x(t)$ on $\mathbb{R}_{\geq 0}$. Equations [1.1] and [1.2] are often simplified by disregarding $w(t)$, and assuming that $y = x$.

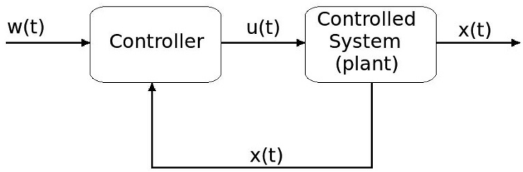

Figure 1.1. *Control/plant model*

An important subclass is the *linear time-invariant* (LTI) framework, for which [1.1] and [1.2] become: $\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ew(t) \\ y(t) = Cx(t) \end{cases}$
for matrices A, B, C, E of appropriate size with constant coefficients. A *discrete-time LTI* system is a system governed by an equation of the form: $x(t+1) = Ax(t) + Bu(t) + Ew(t)$.

When a system is governed by an equation of the form $\dot{x}(t) = Ax(t)$, where A is a matrix whose eigenvalues have negative real parts, the origin is a *stable equilibrium* point to which the system converges from any initial point of $\mathbb{R}^n$. Given a plant governed by an equation of the form $\dot{x}(t) = Ax(t) + Bu(t)$ with $(A, B) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m}$, a typical problem of linear control theory is to find a stabilizing controller governed by an equation of the form $u(t) = Kx(t)$ with $K \in \mathbb{R}^{m\times n}$. This essentially amounts to finding coefficient values of K that make the real parts of the eigenvalues of $A + BK$ negative.